A Comprehensive Mathematical Formalism for Constraint-Based HPSG

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A Logical Language for HPSG

RSRL

The Description Language I

Some examples of traditional HPSG principles:

- The HEAD FEATURE PRINCIPLE
  In a headed phrase, the values of SYNSEM | LOCAL | CATEGORY | HEAD *and* DAUGHTERS | HEAD-DAUGHTER | SYNSEM | LOCAL | CATEGORY | HEAD *are token-identical.*

- The SUBJECT CONDITION (Pollard and Sag, 1994, p. 400)
  If the initial element of the SUBCAT *value of a word is slashed, then so is some other element of that list.*

- The RELATIVE UNIQUENESS PRINCIPLE (Pollard and Sag, 1994, p. 400)
  For any phrase, a member of the set value of SYNSEM | NONLOCAL | INHERITED | REL *may belong to the value of that same path on at most one daughter.*

- Binding Theory, Principle B (Pollard and Sag, 1994, p. 401)
  A personal pronoun must be locally o-free.

A Principle from Abeillé and Godard 1996:

In the original notation:

$$\begin{bmatrix}\text{VFORM} & \textit{part-passé}\\ \text{S-ARG} & \langle \ldots \textit{aff\_r} \ldots \rangle\end{bmatrix} \rightarrow \begin{bmatrix}\text{V-AUX} & \textit{etre}\end{bmatrix}$$

A formalization in RSRL:

$$\left(\begin{array}{l}\exists \boxed{0}\, \exists \boxed{1}\\ \left(\begin{array}{l}\begin{bmatrix}\text{VFORM} & \textit{part-passé}\\ \text{S-ARG} & \boxed{1}\end{bmatrix}\\ \land\, \texttt{member}\left(\boxed{0}[\textit{aff\_r}], \boxed{1}\right)\end{array}\right)\end{array}\right) \rightarrow \begin{bmatrix}\text{V-AUX} & \textit{etre}\end{bmatrix}$$

Signatures

Definition 1. Σ *is a* **signature** *iff*

Σ *is a septuple* $\langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$\langle \mathcal{G}, \sqsubseteq \rangle$ *is a partial order*,

$$\mathcal{S} = \left\{ \sigma \in \mathcal{G} \middle| \begin{array}{l} \text{for each } \sigma' \in \mathcal{G}, \\ \text{if } \sigma' \sqsubseteq \sigma \text{ then } \sigma = \sigma' \end{array} \right\},$$

$\mathcal{A}$ *is a set*,

$\mathcal{F}$ *is a partial function from the Cartesian product of* $\mathcal{G}$ *and* $\mathcal{A}$ *to* $\mathcal{G}$,

for each $\sigma_1 \in \mathcal{G}$, *for each* $\sigma_2 \in \mathcal{G}$ *and for each* $\alpha \in \mathcal{A}$,

if $\mathcal{F}\langle \sigma_1, \alpha \rangle$ *is defined and* $\sigma_2 \sqsubseteq \sigma_1$
then $\mathcal{F}\langle \sigma_2, \alpha \rangle$ *is defined and* $\mathcal{F}\langle \sigma_2, \alpha \rangle \sqsubseteq \mathcal{F}\langle \sigma_1, \alpha \rangle$,

$\mathcal{R}$ *is a finite set, and*

$\mathcal{AR}$ *is a total function from* $\mathcal{R}$ *to* $\mathbb{N}^+$.

Interpretations

Definition 2. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, I *is a* Σ **interpretation** *iff*

I *is a quadruple* $\langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

U *is a set*,

S *is a total function from* U *to* $\mathcal{S}$,

A *is a total function from* $\mathcal{A}$ *to the set of partial functions from* U *to* U,

for each $\alpha \in \mathcal{A}$ *and each* $u \in \mathsf{U}$,

if $\mathsf{A}(\alpha)(u)$ *is defined*
then $\mathcal{F}\langle \mathsf{S}(u), \alpha \rangle$ *is defined, and* $\mathsf{S}(\mathsf{A}(\alpha)(u)) \sqsubseteq \mathcal{F}\langle \mathsf{S}(u), \alpha \rangle$, *and*

for each $\alpha \in \mathcal{A}$ *and each* $u \in \mathsf{U}$,

if $\mathcal{F}\langle \mathsf{S}(u), \alpha \rangle$ *is defined then* $\mathsf{A}(\alpha)(u)$ *is defined*,

R *is a total function from* $\mathcal{R}$ *to the power set of* $\bigcup\limits_{n \in \mathbb{N}} \overline{\mathsf{U}}^n$, *and*

for each $\rho \in \mathcal{R}$, $\mathsf{R}(\rho) \subseteq \overline{\mathsf{U}}^{\mathcal{AR}(\rho)}$.

Chains

Definition 3. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$\widehat{\mathcal{G}} = \mathcal{G} \cup \{chain, echain, nechain, metatop\}$,

$$\widehat{\sqsubseteq} = \sqsubseteq \cup \ \{\langle echain, chain \rangle, \langle nechain, chain \rangle\} \cup \left\{ \langle \sigma, \sigma \rangle \middle| \sigma \in \widehat{\mathcal{G}} \backslash \mathcal{G} \right\} \cup \left\{ \langle \sigma, metatop \rangle \middle| \sigma \in \widehat{\mathcal{G}} \right\},$$

$\widehat{\mathcal{S}} = \mathcal{S} \cup \{echain, nechain\}$, *and*

$\widehat{\mathcal{A}} = \mathcal{A} \cup \{\dagger, \triangleright\}$.

Definition 4. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

$\widehat{\mathsf{S}}$ *is the total function from* $\overline{\mathsf{U}}$ *to* $\widehat{\mathcal{S}}$ *such that*

for each $u \in \mathsf{U}$, $\widehat{\mathsf{S}}(u) = \mathsf{S}(u)$,
for each $u_1 \in \mathsf{U}$, ..., *for each* $u_n \in \mathsf{U}$,

$$\widehat{\mathsf{S}}(\langle u_1, \ldots, u_n \rangle) = \begin{cases} echain & \text{if } n = 0, \\ nechain & \text{if } n > 0 \end{cases}, \text{ and}$$

$\widehat{\mathsf{A}}$ *is the total function from* $\widehat{\mathcal{A}}$ *to the set of partial functions from* $\overline{\mathsf{U}}$ *to* $\overline{\mathsf{U}}$ *such that*

for each $\alpha \in \mathcal{A}$, $\widehat{\mathsf{A}}(\alpha) = \mathsf{A}(\alpha)$,
$\widehat{\mathsf{A}}(\dagger)$ *is the total function from* U^+ *to* U *such that for each* $\langle u_0, \ldots, u_n \rangle \in \mathsf{U}^+$,
$\widehat{\mathsf{A}}(\dagger)(\langle u_0, \ldots, u_n \rangle) = u_0$, *and*
$\widehat{\mathsf{A}}(\triangleright)$ *is the total function from* U^+ *to* U^* *such that for each* $\langle u_0, \ldots, u_n \rangle \in \mathsf{U}^+$,
$\widehat{\mathsf{A}}(\triangleright)(\langle u_0, \ldots, u_n \rangle) = \langle u_1, \ldots, u_n \rangle$.

Formulae

Definition 5. *For each signature Σ, $\mathcal{T}^\Sigma$ and $\mathcal{D}^\Sigma$ are the smallest sets such that*

$: \in \mathcal{T}^\Sigma$,

for each $v \in \mathcal{VAR}$, $v \in \mathcal{T}^\Sigma$,

for each $\alpha \in \widehat{\mathcal{A}}$ *and each* $\tau \in \mathcal{T}^\Sigma$, $\tau\alpha \in \mathcal{T}^\Sigma$,

for each $\sigma \in \widehat{\mathcal{G}}$, *for each* $\tau \in \mathcal{T}^\Sigma$, $\tau \sim \sigma \in \mathcal{D}^\Sigma$,

for each $\tau_1 \in \mathcal{T}^\Sigma$, *for each* $\tau_2 \in \mathcal{T}^\Sigma$, $\tau_1 \approx \tau_2 \in \mathcal{D}^\Sigma$,

for each $\rho \in \mathcal{R}$, *for each* $x_1 \in \mathcal{VAR}$, ..., *for each* $x_{\mathcal{AR}(\rho)} \in \mathcal{VAR}$,

$\rho(x_1, \ldots, x_{\mathcal{AR}(\rho)}) \in \mathcal{D}^\Sigma$,

for each $x \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$, $\exists x\, \delta \in \mathcal{D}^\Sigma$,

for each $x \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$, $\forall x\, \delta \in \mathcal{D}^\Sigma$,

for each $\delta \in \mathcal{D}^\Sigma$, $\neg\delta \in \mathcal{D}^\Sigma$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$, $[\delta_1 \wedge \delta_2] \in \mathcal{D}^\Sigma$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$, $[\delta_1 \vee \delta_2] \in \mathcal{D}^\Sigma$.

The Description Language II

- The TRACE PRINCIPLE (Pollard and Sag, 1994, p. 400)
  The SYNSEM *value of any trace must be a (noninitial) member of the* SUBCAT *list of a substantive word.*

- The CONTROL THEORY (Pollard and Sag, 1994, p. 401)
  (looks for potential controlled *local* objects)
  If the SOA-ARG *value of a control-qfpsoa is token-identical with the* CONTENT *value of a local entity whose* CATEGORY | SUBCAT *value is a list of length one, then the member of that list is (1) reflexive, and (2) coindexed with the* INFLUENCED *(respectively,* COMMITTOR, EXPERIENCER*) value of the control-qfpsoa if the latter is of sort influence (respectively, commitment, orientation).*

- BINDING THEORY
  (looks for potential o-commanders)

- QUANTIFIER BINDING CONDITION (Pollard and Sag, 1994, p. 327)
  Given a quantifier contained within a CONTENT *value, every occurrence within that* CONTENT *value of the quantifier's index must be captured by the quantifier.*

Formula Denotation

Definition 6. *For each signature Σ, for each Σ interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

$Ass_{\mathsf{I}} = \overline{\mathsf{U}}^{\mathcal{VAR}}$ *is the* ***set of variable assignments in*** I.

Definition 7. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each Σ interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $ass \in Ass_{\mathsf{I}}$, T^{ass}_{I} *is the total function from* $\mathcal{T}^\Sigma$ *to the set of partial functions from* U *to* $\overline{\mathsf{U}}$ *such that for each* $u \in \mathsf{U}$,

$T^{ass}_{\mathsf{I}}(:)(u)$ *is defined and* $T^{ass}_{\mathsf{I}}(:)(u) = u$,

for each $v \in \mathcal{VAR}$, $T^{ass}_{\mathsf{I}}(v)(u)$ *is defined and* $T^{ass}_{\mathsf{I}}(v)(u) = ass(v)$,

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\alpha \in \widehat{\mathcal{A}}$,

$T^{ass}_{\mathsf{I}}(\tau\alpha)(u)$ *is defined*
iff $T^{ass}_{\mathsf{I}}(\tau)(u)$ *is defined and* $\widehat{\mathsf{A}}(\alpha)(T^{ass}_{\mathsf{I}}(\tau)(u))$ *is defined, and*
if $T^{ass}_{\mathsf{I}}(\tau\alpha)(u)$ *is defined then* $T^{ass}_{\mathsf{I}}(\tau\alpha)(u) = \widehat{\mathsf{A}}(\alpha)(T^{ass}_{\mathsf{I}}(\tau)(u))$.

Definition 8. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *and for each* $u \in \mathsf{U}$,

$$\mathsf{Co}^u_{\mathsf{I}} = \left\{ u' \in \mathsf{U} \;\middle|\; \begin{array}{l} \text{for some } ass \in Ass_{\mathsf{I}}, \\ \text{for some } \pi \in \mathcal{A}^*, \\ \quad T^{ass}_{\mathsf{I}}(:\pi)(u) \text{ is defined, and} \\ \quad u' = T^{ass}_{\mathsf{I}}(:\pi)(u) \end{array} \right\}.$$

Definition 9. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $ass \in Ass_{\mathsf{I}}$, *for each* $v \in \mathcal{VAR}$, *for each* $w \in \mathcal{VAR}$, *for each* $u \in \overline{\mathsf{U}}$,

$$ass\tfrac{u}{v}(w) = \begin{cases} u & \text{if } v = w \\ ass(w) & \text{otherwise.} \end{cases}$$

Definition 10. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $ass \in Ass_{\mathsf{I}}$, D^{ass}_{I} *is the total function from* $\mathcal{D}^\Sigma$ *to the power set of* U *such that*

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\sigma \in \widehat{\mathcal{G}}$,

$$D^{ass}_{\mathsf{I}}(\tau \sim \sigma) = \left\{ u \in \mathsf{U} \;\middle|\; \begin{array}{l} T^{ass}_{\mathsf{I}}(\tau)(u) \text{ is defined, and} \\ \widehat{\mathsf{S}}(T^{ass}_{\mathsf{I}}(\tau)(u)) \mathrel{\widehat{\sqsubseteq}} \sigma \end{array} \right\},$$

for each $\tau_1 \in \mathcal{T}^\Sigma$, *for each* $\tau_2 \in \mathcal{T}^\Sigma$,

$$D^{ass}_{\mathsf{I}}(\tau_1 \approx \tau_2) = \left\{ u \in \mathsf{U} \;\middle|\; \begin{array}{l} T^{ass}_{\mathsf{I}}(\tau_1)(u) \text{ is defined,} \\ T^{ass}_{\mathsf{I}}(\tau_2)(u) \text{ is defined, and} \\ T^{ass}_{\mathsf{I}}(\tau_1)(u) = T^{ass}_{\mathsf{I}}(\tau_2)(u) \end{array} \right\},$$

for each $\rho \in \mathcal{R}$, *for each* $x_1 \in \mathcal{VAR}$, ..., *for each* $x_{\mathcal{AR}(\rho)} \in \mathcal{VAR}$,

$$D^{ass}_{\mathsf{I}}(\rho(x_1, \ldots, x_{\mathcal{AR}(\rho)})) = \left\{ u \in \mathsf{U} \;\middle|\; \left\langle ass(x_1), \ldots, ass(x_{\mathcal{AR}(\rho)}) \right\rangle \in \mathsf{R}(\rho) \right\},$$

for each $v \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$,

$$D^{ass}_{\mathsf{I}}(\exists v\ \delta) = \left\{ u \in \mathsf{U} \;\middle|\; \begin{array}{l} \text{for some } u' \in \overline{\mathsf{Co}^u_{\mathsf{I}}}, \\ u \in D^{ass\frac{u'}{v}}_{\mathsf{I}}(\delta) \end{array} \right\},$$

for each $v \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$,

$$D^{ass}_{\mathsf{I}}(\forall v\ \delta) = \left\{ u \in \mathsf{U} \;\middle|\; \begin{array}{l} \text{for each } u' \in \overline{\mathsf{Co}^u_{\mathsf{I}}}, \\ u \in D^{ass\frac{u'}{v}}_{\mathsf{I}}(\delta) \end{array} \right\},$$

for each $\delta \in \mathcal{D}^\Sigma$,

$$D^{ass}_{\mathsf{I}}(\neg\delta) = \mathsf{U} \backslash D^{ass}_{\mathsf{I}}(\delta),$$

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$D^{ass}_{\mathsf{I}}([\delta_1 \wedge \delta_2]) = D^{ass}_{\mathsf{I}}(\delta_1) \cap D^{ass}_{\mathsf{I}}(\delta_2),$$

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$D^{ass}_{\mathsf{I}}([\delta_1 \vee \delta_2]) = D^{ass}_{\mathsf{I}}(\delta_1) \cup D^{ass}_{\mathsf{I}}(\delta_2), \text{ and}$$

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$D^{ass}_{\mathsf{I}}([\delta_1 \rightarrow \delta_2]) = (\mathsf{U} \backslash D^{ass}_{\mathsf{I}}(\delta)) \cup D^{ass}_{\mathsf{I}}(\delta_2).$$

From Formulae to Descriptions

Definition 11. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$FV(:) = \emptyset$,

for each $v \in \mathcal{VAR}$, $FV(v) = \{v\}$,

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\alpha \in \widehat{\mathcal{A}}$, $FV(\tau\alpha) = FV(\tau)$,

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\sigma \in \widehat{\mathcal{G}}$, $FV(\tau \sim \sigma) = FV(\tau)$,

for each $\tau_1, \tau_2 \in \mathcal{T}^\Sigma$, $FV(\tau_1 \approx \tau_2) = FV(\tau_1) \cup FV(\tau_2)$,

for each $\rho \in \mathcal{R}$, *for each* $x_1, \ldots, x_{\mathcal{AR}(\rho)} \in \mathcal{VAR}$,

$$FV(\rho(x_1, \ldots, x_{\mathcal{AR}(\rho)})) = \{x_1, \ldots, x_{\mathcal{AR}(\rho)}\},$$

for each $\delta \in \mathcal{D}^\Sigma$, *for each* $v \in \mathcal{VAR}$, $FV(\exists v\ \delta) = FV(\delta) \backslash \{v\}$,

for each $\delta \in \mathcal{D}^\Sigma$, *for each* $v \in \mathcal{VAR}$, $FV(\forall v\ \delta) = FV(\delta) \backslash \{v\}$,

for each $\delta \in \mathcal{D}^\Sigma$, $FV(\neg\delta) = FV(\delta)$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$FV([\delta_1 \wedge \delta_2]) = FV(\delta_1) \cup FV(\delta_2)$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$FV([\delta_1 \vee \delta_2]) = FV(\delta_1) \cup FV(\delta_2)$, *and*

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$FV([\delta_1 \rightarrow \delta_2]) = FV(\delta_1) \cup FV(\delta_2)$.

Proposition 1. *For each signature* Σ, *for each* Σ *interpretation* I, *for each* $ass_1 \in Ass_\mathsf{I}$, *for each* $ass_2 \in Ass_\mathsf{I}$,

for each $\tau \in \mathcal{T}^\Sigma$,

if for each $v \in FV(\tau)$, $ass_1(v) = ass_2(v)$
then $T_\mathsf{I}^{ass_1}(\tau) = T_\mathsf{I}^{ass_2}(\tau)$, *and*

for each $\delta \in \mathcal{D}^\Sigma$,

if for each $v \in FV(\delta)$, $ass_1(v) = ass_2(v)$
then $D_\mathsf{I}^{ass_1}(\delta) = D_\mathsf{I}^{ass_2}(\delta)$.

Definition 12. *For each signature* Σ,

$$\mathcal{D}_0^\Sigma = \{\delta \in \mathcal{D}^\Sigma \mid FV(\delta) = \emptyset\}.$$

Corollary 1. *For each signature* Σ, *for each* $\delta \in \mathcal{D}_0^\Sigma$, *for each* Σ *interpretation* I, *for each* $ass_1 \in Ass_\mathsf{I}$, *for each* $ass_2 \in Ass_\mathsf{I}$,

$$D_\mathsf{I}^{ass_1}(\delta) = D_\mathsf{I}^{ass_2}(\delta).$$

Description Denotation and Theory Denotation

Definition 13. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, D_I *is the total function from* $\mathcal{D}_0^\Sigma$ *to the power set of* U *such that for each* $\delta \in \mathcal{D}_0^\Sigma$,

$$D_\mathsf{I}(\delta) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} \textit{for each } ass \in Ass_\mathsf{I}, \\ u \in D_\mathsf{I}^{ass}(\delta) \end{array} \right\}.$$

Definition 14. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, Θ_I *is the total function from the power set of* $\mathcal{D}_0^\Sigma$ *to the power set of* U *such that for each* $\theta \subseteq \mathcal{D}_0^\Sigma$,

$$\Theta_\mathsf{I}(\theta) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} \textit{for each } \delta \in \theta, \\ u \in D_\mathsf{I}(\delta) \end{array} \right\}.$$

Grammars and Models

Definition 15. Γ *is a* ***grammar*** *iff*

Γ *is a pair* $\langle \Sigma, \theta \rangle$,

Σ *is a signature, and*

$\theta \subseteq \mathcal{D}_0^\Sigma$.

Definition 16. *For each grammar* $\Gamma = \langle \Sigma, \theta \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

I *is a* Γ ***model*** *iff* $\Theta_\mathsf{I}(\theta) = \mathsf{U}$.

King 1999 in RSRL

An Example

A Signature:

top

$\quad car \quad$ OWNER $\quad person$
$\quad\quad\quad\quad$ DRIVER $\quad person$

$\quad\quad vw$

$\quad\quad bmw$

$\quad person \quad$ LIKES-BEST $\quad top$

$\quad\quad man$

$\quad\quad woman$

$\mathcal{R} = \{\}$, $\mathcal{AR} = \{\}$

An interpretation of our signature:
Let $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, with:
$\mathsf{U} = \left\{ \begin{array}{l} \text{Anke, Anke's husband, Detmar, Kordula,} \\ \text{first car, second car} \end{array} \right\}$
S(Anke) $= woman$,
S(Kordula) $= woman$,
S(Anke's husband) $= man$,
S(Detmar) $= man$,
S(first car) $= vw$, and
S(second car) $= vw$
A(LIKES-BEST)(Anke) = Anke's husband,
A(LIKES-BEST)(Anke's husband) = Anke,
A(LIKES-BEST)(Detmar) = Kordula,
A(LIKES-BEST)(Kordula) = Detmar,
A(OWNER)(first car) = Kordula,
A(OWNER)(second car) = Anke,
A(DRIVER)(first car) = Detmar,
A(DRIVER)(second car) = Anke,
A(LIKES-BEST) is undefined on the first car and on the second car,
A(OWNER) is undefined on Anke, Anke's husband, Detmar and Kordula, and
A(DRIVER) is likewise undefined on Anke, Anke's husband, Detmar and Kordula, and
R $= \{\}$.

Grammars of I:

Assume $\theta_1 = \left\{ \begin{array}{l} [: \sim bmw \rightarrow \neg : \approx :], \\ [: \sim vw \rightarrow : \text{OWNER} \sim woman] \end{array} \right\}$.

Observation:
I is a model of $\langle \Sigma, \theta_1 \rangle$. However, $\langle \Sigma, \theta_1 \rangle$ overlicenses I.
I_2 is a relevant example:

Let $\mathsf{I}_2 = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, with:
U = {Anne, Manfred, Maria, Mike, third car}
S(Anne) $= woman$,
S(Maria) $= woman$,
S(Manfred) $= man$,
S(Mike) $= man$,
S(third car) $= vw$, and
A(LIKES-BEST)(Anne) = Manfred,
A(LIKES-BEST)(Manfred) = Anne,
A(LIKES-BEST)(Maria) = Mike,
A(LIKES-BEST)(Mike) = Maria,
A(OWNER)(third car) = Anne,
A(DRIVER)(third car) = Mike,
A(LIKES-BEST) is undefined on the third car,
A(OWNER) is undefined on Anne, Manfred, Maria and Mike, and

A(DRIVER) is likewise undefined on Anne, Manfred, Maria and Mike, and
$\mathsf{R} = \{\}$.

Now assume

$$\theta_2 = \left\{ \begin{array}{l} [: \sim woman \leftrightarrow [: \textsc{likes-best} \sim man \wedge : \textsc{likes-best likes-best} \approx :]] , \\ [: \sim man \leftrightarrow [: \textsc{likes-best} \sim woman \wedge : \textsc{likes-best likes-best} \approx :]] , \\ [: \sim bmw \rightarrow \neg : \approx :] , \\ [: \sim vw \rightarrow [: \textsc{owner} \sim woman \wedge : \textsc{owner} \approx : \textsc{driver}]] \end{array} \right\}.$$

Problem:
$\langle \Sigma, \theta_2 \rangle$ underlicenses I. I is not even a model of $\langle \Sigma, \theta_2 \rangle$.

Finally, consider

$$\theta_3 = \left\{ \begin{array}{l} [: \sim woman \leftrightarrow [: \textsc{likes-best} \sim man \wedge : \textsc{likes-best likes-best} \approx :]] , \\ [: \sim man \leftrightarrow [: \textsc{likes-best} \sim woman \wedge : \textsc{likes-best likes-best} \approx :]] , \\ [: \sim bmw \rightarrow \neg : \approx :] , \\ \left[: \sim vw \rightarrow \left[\begin{array}{l} : \textsc{owner} \sim woman \wedge \\ [: \textsc{owner} \approx : \textsc{driver} \vee : \textsc{driver} \approx : \textsc{owner likes-best}] \end{array} \right] \right] \end{array} \right\}.$$

Observation:
I is an exhaustive model of $\langle \Sigma, \theta_3 \rangle$.

Formalization of a few principles of Pollard and Sag 1994

Throughout the following examples, the syntax used in the formalization of principles of grammar is the AVM notation for RSRL defined in Section 3.2 of Richter 2000.

(1) *The* SUBJECT CONDITION *(Pollard and Sag, 1994, p. 400)*

If the initial element of the SUBCAT value of a word is slashed, then so is some other element of that list.

(2) *The* SUBJECT CONDITION *formalized*

$$\forall \boxed{1} \left(\begin{bmatrix} word \\ \textsc{ss loc cat subcat} \ \left\langle [\textsc{nonloc inherited slash} \ neset] \,|\, \boxed{1} \right\rangle \end{bmatrix} \rightarrow \exists x \ \mathtt{member}({}^{x}[\textsc{nonloc inherited slash} \ neset], \boxed{1}) \right)$$

(3) *The* CONTROL THEORY *(Pollard and Sag, 1994, p. 401)*

If the SOA-ARG value of a *control-qfpsoa* is token-identical with the CONTENT value of a *local* entity whose CATEGORY | SUBCAT value is a list of length one, then the member of that list is (1) reflexive, and (2) coindexed with the INFLUENCED (respectively, COMMITTOR, EXPERIENCER) value of the *control-qfpsoa* if the latter is of sort *influence* (respectively, *commitment*, *orientation*).

(4) *The* CONTROL THEORY *formalized*

$$\forall \boxed{1} \forall \boxed{2} \forall \boxed{3} \left(\begin{array}{l} \exists \boxed{4} \left(\boxed{1} \begin{bmatrix} influence \\ \textsc{soa-arg} \ \boxed{4} \end{bmatrix} \wedge \boxed{2} \begin{bmatrix} local \\ \textsc{cat subcat} \ \langle \boxed{3} \rangle \\ \textsc{cont} \ \boxed{4} \end{bmatrix} \right) \rightarrow \\ \exists \boxed{5} \left(\boxed{3} \left[\textsc{loc cont} \begin{bmatrix} reflexive \\ \textsc{index} \ \boxed{5} \end{bmatrix} \right] \wedge \boxed{1} [\textsc{influenced} \ \boxed{5}] \right) \end{array} \right) \wedge$$

$$\forall \boxed{1} \forall \boxed{2} \forall \boxed{3} \left(\begin{array}{l} \exists \boxed{4} \left(\boxed{1} \begin{bmatrix} commitment \\ \textsc{soa-arg} \ \boxed{4} \end{bmatrix} \wedge \boxed{2} \begin{bmatrix} local \\ \textsc{cat subcat} \ \langle \boxed{3} \rangle \\ \textsc{cont} \ \boxed{4} \end{bmatrix} \right) \rightarrow \\ \exists \boxed{5} \left(\boxed{3} \left[\textsc{loc cont} \begin{bmatrix} reflexive \\ \textsc{index} \ \boxed{5} \end{bmatrix} \right] \wedge \boxed{1} [\textsc{committor} \ \boxed{5}] \right) \end{array} \right) \wedge$$

$$\forall \boxed{1} \forall \boxed{2} \forall \boxed{3} \left(\begin{array}{l} \exists \boxed{4} \left(\boxed{1} \begin{bmatrix} orientation \\ \textsc{soa-arg} \ \boxed{4} \end{bmatrix} \wedge \boxed{2} \begin{bmatrix} local \\ \textsc{cat subcat} \ \langle \boxed{3} \rangle \\ \textsc{cont} \ \boxed{4} \end{bmatrix} \right) \rightarrow \\ \exists \boxed{5} \left(\boxed{3} \left[\textsc{loc cont} \begin{bmatrix} reflexive \\ \textsc{index} \ \boxed{5} \end{bmatrix} \right] \wedge \boxed{1} [\textsc{orientation} \ \boxed{5}] \right) \end{array} \right)$$

(5) *The* BINDING THEORY *(Pollard and Sag, 1994, p. 401)*

Principle A. A locally o-commanded anaphor must be locally o-bound.
Principle B. A personal pronoun must be locally o-free.
Principle C. A nonpronoun must be o-free.

The BINDING THEORY employs a number of auxiliary relations that we must define first. For constraining the meaning of relation symbols in the grammar, we employ a convenient clause notation.

> "One $synsem$ object is *more oblique* than another provided it appears to the right of the other on the SUBCAT list of some word." (Pollard and Sag, 1994, p. 401)

(6) $\texttt{more-oblique}(x, y) \overset{\forall}{\Longleftarrow}$

$$ {}^{w}\begin{bmatrix} \textit{word} \\ \text{SS LOC CAT SUBCAT } \boxed{1} \end{bmatrix} \land \texttt{to-the-right}(x, y, \boxed{1}) $$

(7) $\texttt{to-the-right}(x, y, z) \overset{\forall}{\Longleftarrow}$

$$ \begin{aligned} & \exists \boxed{1}\ (z \langle y | \boxed{1} \rangle \land \texttt{member}(x, \boxed{1})) \\ & \lor \exists \boxed{1}\ (z \langle \textit{object} | \boxed{1} \rangle \land \texttt{to-the-right}(x, y, \boxed{1})) \end{aligned} $$

> "One referential $synsem$ object *locally o-commands* another provided they have distinct LOCAL values and either (1) the second is more oblique than the first, or (2) the second is a member of the SUBCAT list of a $synsem$ object that is more oblique than the first."

(8) $\texttt{loc-o-command}(x, y) \overset{\forall}{\Longleftarrow}$

$$ \begin{pmatrix} {}^{x}\Big[\text{LOC } \boxed{1} \big[\text{CONT INDEX } \textit{ref}\big]\Big] \land \\ {}^{y}\Big[\text{LOC } \boxed{2} \big[\text{CONT INDEX } \textit{ref}\big]\Big] \land \\ \neg \boxed{1} = \boxed{2} \end{pmatrix} \land \begin{pmatrix} \texttt{more-oblique}(y, x) \lor \\ \begin{pmatrix} \texttt{more-oblique}\left({}^{s}\begin{bmatrix} \textit{synsem} \\ \text{LOC CAT SUBCAT } \boxed{3} \end{bmatrix}, x \right) \land \\ \texttt{member}(y, \boxed{3}) \end{pmatrix} \end{pmatrix} $$

> "One referential $synsem$ object *o-commands* another provided they have distinct LOCAL values and either (1) the second is more oblique than the first, (2) the second is a member of the SUBCAT list of a $synsem$ object that is o-commanded by the first, or (3) the second has the same LOCAL | CATEGORY | HEAD value as a $synsem$ object that is o-commanded by the first." (Pollard and Sag, 1994, p. 401)

(9) $\texttt{o-command}(x, y) \overset{\forall}{\Longleftarrow}$

$$ \begin{pmatrix} {}^{x}\Big[\text{LOC } \boxed{1} \big[\text{CONT INDEX } \textit{ref}\big]\Big] \land \\ {}^{y}\left[\text{LOC } \boxed{2} \begin{bmatrix} \text{CAT HEAD } \boxed{4} \\ \text{CONT INDEX } \textit{ref} \end{bmatrix}\right] \land \\ \neg \boxed{1} = \boxed{2} \end{pmatrix} \land \begin{pmatrix} \texttt{more-oblique}(y, x) \lor \\ \begin{pmatrix} \texttt{o-command}\left(x, {}^{s_1}\begin{bmatrix} \textit{synsem} \\ \text{LOC CAT SUBCAT } \boxed{3} \end{bmatrix}\right) \land \\ \texttt{member}(y, \boxed{3}) \end{pmatrix} \lor \\ \texttt{o-command}\left(x, {}^{s_2}\begin{bmatrix} \textit{synsem} \\ \text{LOC } \big[\text{CAT HEAD } \boxed{4}\big] \end{bmatrix}\right) \end{pmatrix} $$

> "One referential $synsem$ object *(locally) o-binds* another provided it (locally) o-commands and is coindexed with the other. A referential $synsem$ object is *(locally) o-free* provided it is not (locally) o-bound. Two $synsem$ entities are *coindexed* provided their LOCAL | CONTENT | INDEX values are token-identical." (Pollard and Sag, 1994, p. 401)

(10) $\texttt{loc-o-bind}(x, y) \overset{\forall}{\Longleftarrow}$

$$ \texttt{loc-o-command}\big({}^{x}[\text{LOC CONT INDEX } \boxed{1}],\ {}^{y}[\text{LOC CONT INDEX } \boxed{1}] \big) $$

(11) $\texttt{o-bind}(x, y) \overset{\forall}{\Longleftarrow}$

$$ \texttt{o-command}\big({}^{x}[\text{LOC CONT INDEX } \boxed{1}],\ {}^{y}[\text{LOC CONT INDEX } \boxed{1}] \big) $$

(12) *The* BINDING THEORY *formalized*

Principle A:

$$\forall x \left(\begin{array}{l} \exists y\, \texttt{loc-o-command}(y,\ {}^{x}[\text{LOC CONT } ana]) \rightarrow \\ \exists z\, \texttt{loc-o-bind}(z, x) \end{array} \right)$$

Principle B:

$$\forall x\ (\ {}^{x}[\text{LOC CONT } ppro] \rightarrow \neg\exists y\ \texttt{loc-o-bind}(y, x))$$

Principle C:

$$\forall x\ (\ {}^{x}[\text{LOC CONT } npro] \rightarrow \neg\exists y\ \texttt{o-bind}(y, x))$$

Terminology

Definition 17. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $u \in \mathsf{I}$,

$\mathsf{I}_u = \langle \mathsf{U}_u, \mathsf{S}_u, \mathsf{A}_u, \mathsf{R}_u \rangle$ *is the* Σ ***interpretation under*** u ***in*** I *iff*

$\mathsf{U}_u = \left\{ u' \in \mathsf{U} \,\middle|\, u' \in \mathsf{Co}_{\mathsf{I}}^{u} \right\}$,
$\mathsf{S}_u = \mathsf{S} \cap (\mathsf{U}_u \times \mathcal{S})$,
$\mathsf{A}_u = \mathsf{A} \cap (\mathcal{A} \times \{\mathsf{U}_u \times \mathsf{U}_u\})$,
$\mathsf{R}_u = \mathsf{R} \cap \left(\mathcal{R} \times Pow \left(\bigcup_{n \in \mathbb{N}} \left(\overline{\mathsf{U}}_u\right)^n \right) \right)$, *and*
$\mathsf{I}_u = \langle \mathsf{U}_u, \mathsf{S}_u, \mathsf{A}_u, \mathsf{R}_u \rangle$.

Definition 18. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $u \in \mathsf{I}$,

$\langle u, \mathsf{I}_u \rangle$ *is the* ***configuration of entities under*** u ***in*** I *iff*

I_u *is the* Σ *interpretation under* u *in* I.

Definition 19. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1 \rangle$, *for each* $u_1 \in \mathsf{U}_1$, *for each* Σ *interpretation* $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2 \rangle$, *for each* $u_2 \in \mathsf{U}_2$,

f *is a* ***congruence*** *from* $\langle u_1, \mathsf{I}_1 \rangle$ *to* $\langle u_2, \mathsf{I}_2 \rangle$ *in* Σ *iff*

f is a bijection from $\overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$ *to* $\overline{\mathsf{Co}_{\mathsf{I}_2}^{u_2}}$,
for each $u \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$, $\widehat{\mathsf{S}_1}(u) = \widehat{\mathsf{S}_2}(f(u))$,
for each $\alpha \in \widehat{\mathcal{A}}$, *for each* $u \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$,
$\widehat{\mathsf{A}_1}(\alpha)(u)$ *is defined iff* $\widehat{\mathsf{A}_2}(\alpha)(f(u))$ *is defined, and*
if $\widehat{\mathsf{A}_1}(\alpha)(u)$ *is defined then* $f(\widehat{\mathsf{A}_1}(\alpha)(u)) = \widehat{\mathsf{A}_2}(\alpha)(f(u))$,
for each $\rho \in \mathcal{R}$, *for each* $o_1 \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}, \ldots,$ *for each* $o_{\mathcal{AR}(\rho)} \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$,
$\langle o_1, \ldots, o_{\mathcal{AR}(\rho)} \rangle \in \mathsf{R}_1(\rho)$
iff $\langle f(o_1), \ldots, f(o_{\mathcal{AR}(\rho)}) \rangle \in \mathsf{R}_2(\rho)$, *and*
$f(u_1) = u_2$.

Definition 20. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1 \rangle$, *for each* $u_1 \in \mathsf{U}_1$, *for each* Σ *interpretation* $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2 \rangle$, *for each* $u_2 \in \mathsf{U}_2$,

$\langle u_1, \mathsf{I}_1 \rangle$ *and* $\langle u_2, \mathsf{I}_2 \rangle$ *are* ***congruent in*** Σ *iff for some* f, f *is a congruence from* $\langle u_1, \mathsf{I}_1 \rangle$ *to* $\langle u_2, \mathsf{I}_2 \rangle$ *in* Σ.

Linguistic Truth in King 1999

The three conditions of King 1999, p. 343, can now straightforwardly be cast in terms of RSRL. The only difference to the original three conditions is that I am claiming that natural languages include relations between components (and chains of components) of tokens. A grammar $\langle\Sigma, \theta\rangle$ is true of a natural language only if

1. the natural language can be construed as an interpretation, I, of the signature Σ, where I is a system of linguistic tokens,

2. each description in θ is true of each entity in the natural language, i.e., the natural language, I, is a $\langle\Sigma, \theta\rangle$ model, and

3. some description in θ is false of some component of each entity, u', in any Σ interpretation, I$'$, for which no entity u in the natural language, I, has isomorphically configured components.

Except for the reference to the extended definitions of all relevant terms, the conditions 1–3 are identical to

King's conditions on SRL grammars. The third condition amounts to saying that the language that is described by a grammar is in the class of exhaustive models of the grammar. Since we know that for each grammar, there exists an exhaustive model, it follows that King's interpretation of the meaning of a grammar is fully compatible with RSRL.

Exhaustive Models

Based on the notion of congruence, we can give an algebraic characterization of exhaustive models:

Definition 21. *For each signature Σ, for each Σ interpretation $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1\rangle$, for each Σ interpretation $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2\rangle$,*

*I_1 **simulates** I_2 **in** Σ iff*
for each $u_2 \in \mathsf{U}_2$, for some $u_1 \in \mathsf{U}_1$, $\langle u_1, \mathsf{I}_1\rangle$ and $\langle u_2, \mathsf{I}_2\rangle$ are congruent in Σ.

Definition 22. *For each signature Σ, for each $\theta \subseteq \mathcal{D}_0^{\Sigma}$, for each Σ interpretation I,*

*I is an **exhaustive** $\langle\Sigma, \theta\rangle$ **model** iff*

I is a $\langle\Sigma, \theta\rangle$ model, and
for each Σ interpretation I$'$,
if I$'$ is a $\langle\Sigma, \theta\rangle$ model then I simulates I$'$ in Σ.

A simpler characterization of exhaustive models can be based on description denotations alone:

Definition 23. *For each signature Σ, for each Σ interpretation $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1\rangle$, for each $u_1 \in \mathsf{U}_1$, for each Σ interpretation $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2\rangle$, for each $u_2 \in \mathsf{U}_2$,*

*$\langle u_1, \mathsf{I}_1\rangle$ and $\langle u_2, \mathsf{I}_2\rangle$ are **indiscernible in** Σ*
iff for each $\delta \in \mathcal{D}_0^{\Sigma}$, $u_1 \in D_{\mathsf{I}_1}(\delta)$ iff $u_2 \in D_{\mathsf{I}_2}(\delta)$.

Indiscernibility and congruence turn out to be equivalent:

Proposition 2. *For each signature Σ, for each Σ interpretation $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1\rangle$, for each $o_1 \in \mathsf{U}_1$, for each Σ interpretation $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2\rangle$, for each $o_2 \in \mathsf{U}_2$,*

$\langle o_1, \mathsf{I}_1\rangle$ and $\langle o_2, \mathsf{I}_2\rangle$ are congruent in Σ iff $\langle o_1, \mathsf{I}_1\rangle$ and $\langle o_2, \mathsf{I}_2\rangle$ are indiscernible in Σ.

Since indiscernibility and congruence are equivalent, the simulation of models of grammars can alternatively be expressed in terms of the denotation of sets of descriptions in interpretations of the same signature.

One Σ model I of a grammar, $\langle \Sigma, \theta \rangle$, simulates another Σ model I′ of the grammar in Σ only if each Σ theory that describes something in I′ also describes something in I.

Proposition 3. *For each signature Σ, for each $\theta \subseteq \mathcal{D}_0^\Sigma$, for each Σ interpretation I,*

for each Σ interpretation I′,

if I′ is a $\langle \Sigma, \theta \rangle$ model then I simulates I′ in Σ

iff for each $\theta' \subseteq \mathcal{D}_0^\Sigma$, for each Σ interpretation I′,

if I′ is a $\langle \Sigma, \theta \rangle$ model and $\Theta_{I'}(\theta') \neq \emptyset$ then $\Theta_I(\theta') \neq \emptyset$.

Corollary 2. *For each grammar $\Gamma = \langle \Sigma, \theta \rangle$, for each Σ interpretation I,*

I is an exhaustive Γ model iff

I is a Γ model, and
for each $\theta' \subseteq \mathcal{D}_0^\Sigma$, for each Σ interpretation I′,
if I′ is a Γ model and $\Theta_{I'}(\theta') \neq \emptyset$ then $\Theta_I(\theta') \neq \emptyset$.

The Exhaustive Models Theorem of RSRL

Theorem 1. *For each signature Σ, for each Σ theory θ, there exists a Σ interpretation I such that I is an exhaustive $\langle \Sigma, \theta \rangle$ model.*

As a corollary of THEOREM 1, we get immediately that if an RSRL grammar has a nonempty model, it also has a nonempty exhaustive model. In summation, a non-vacuous grammar has a linguistically sensible meaning in the class of its exhaustive models. King's explanation of the meaning of grammars, *mutatis mutandis*, carries over to RSRL.

A proof of THEOREM 1, due to Richter and King 1997, can be found in Appendix A of Richter 2000.

Pollard 1999 in RSRL

Pollard Feature Structures

On page 34, we introduced the notions of a "Σ interpretation under u in I, I_u" and of a "configuration of entities under u in I, $\langle u, I_u \rangle$". For each signature Σ, for each Σ interpretation I, and for each $u \in I$, Pollard 1999 calls the counterpart in SRL of I_u a Σ *interpretation generated by u in* I; and, for each signature Σ, for each Σ interpretation I, and for each $u \in I$, Pollard 1999 calls the SRL counterpart of $\langle u, I_u \rangle$ a *feature structure* (determined by u in I). Since these mathematical structures differ from standard feature structures in important ways, and since we are concerned with a structural extension of Pollard's entities, I call $\langle u, I_u \rangle$ *extended Pollard feature structures*.

According to their definition, the extended Pollard feature structures are Pollard's analogue to King's linguistic tokens. To obtain his unique representational idealizations of isomorphic utterance tokens, Pollard must provide a canonical representation of isomorphic utterance tokens. It is the purpose of the following definition of a *node abstraction* to provide the necessary prerequisites:

Definition 24. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, ν *is a* **node abstraction in** Σ *iff*

ν *is a pair* $\langle \beta, \varrho \rangle$,

$\beta \subseteq \mathcal{T}^{\Sigma}$,

$: \in \beta$,

for each $\tau \in \mathcal{T}^{\Sigma}$, *for each* $\varphi \in \mathcal{A}$, *if* $\tau\varphi \in \beta$ *then* $\tau \in \beta$,

ϱ *is an equivalence relation over* β,

for each $\tau_1 \in \mathcal{T}^{\Sigma}$, *for each* $\tau_2 \in \mathcal{T}^{\Sigma}$, *for each* $\varphi \in \mathcal{A}$,

if $\tau_1\varphi \in \beta$ *and* $\langle \tau_1, \tau_2 \rangle \in \varrho$ *then* $\langle \tau_1\varphi, \tau_2\varphi \rangle \in \varrho$.

β is a prefix closed set of terms that contains at least the colon, and ϱ is a right invariant equivalence relation over β.

Extended Pollard abstract feature structures are canonical representations of extended Pollard feature structures. Their nodes are equivalence classes of terms in β, and their root entities are in the equivalence class $|:|_{\varrho}$:

Definition 25. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $u \in \mathsf{U}$,

$\langle u, \mathsf{I} \rangle$ *is an* **extended Pollard abstract feature structure under** Σ *iff*

$\langle u, \mathsf{I} \rangle$ *is an extended Pollard feature structure,*
for some node abstraction in Σ, $\langle \beta, \varrho \rangle$,
$\mathsf{U} = Quo(\langle \beta, \varrho \rangle)$,
$u = |:|_{\varrho}$,
for each $u_1 \in Quo(\langle \beta, \varrho \rangle)$, *for each* $u_2 \in Quo(\langle \beta, \varrho \rangle)$, *for each* $\varphi \in \mathcal{A}$,
$\mathsf{A}(\varphi)(u_1) = u_2$ *iff*
for some $\tau \in \beta$, $u_1 = |\tau|_{\varrho}$, *and* $u_2 = |\tau\varphi|_{\varrho}$.

I write $\mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$ for the set of extended Pollard abstract feature structures under Σ. We obtain the following crucial result:

Proposition 4. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $v \in \mathsf{U}$, *if* $\langle v, \mathsf{I} \rangle$ *is an extended Pollard feature structure then there exists exactly one* $\langle v', \mathsf{I}' \rangle \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$, *such that for each* $\delta \in \mathcal{D}^{\Sigma}$,

$v \in D_{\mathsf{I}}(\delta)$ *iff* $v' \in D_{\mathsf{I}'}(\delta)$.

Strong Generative Capacity

We start with an auxiliary definition:

Definition 26. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $v \in \mathsf{U}$,

$\langle v, \mathsf{I}_v \rangle$ *is the* **extended Pollard feature structure determined by** v **in** I *iff*

I_v *is the* Σ *interpretation generated by* v *in* I.

Pollard 1999 presents three equivalent definitions of the strong generative capacity of a grammar. All of them use the familiar relationships between the models of a grammar, the class of exhaustive models of a grammar, and the set of abstract feature structures that can be abstracted from them.

For the first definition of the strong generative capacity of a grammar, Pollard defines an abstraction function, $\mathrm{Abst}_{\mathsf{I}}$. For each entity v in the interpretation I, it contains pairs of v and extended Pollard abstract feature structures whose root entities are indiscernible from v:

Definition 27. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

$\mathrm{Abst}_{\mathsf{I}}$ *is the binary relation on* $\mathsf{U} \times \mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$ *such that, for each* $v \in \mathsf{U}$, *for each* $\langle v', \mathsf{I}' \rangle \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$, $\langle v, \langle v', \mathsf{I}' \rangle \rangle \in \mathrm{Abst}_{\mathsf{I}}$ *iff*

for the extended Pollard feature structure $\langle v, \mathsf{I}_v \rangle$ *determined by* v *in* I, *for each* $\delta \in \mathcal{D}^{\Sigma}$, $v \in D_{\mathsf{I}_v}(\delta)$ *iff* $v' \in D_{\mathsf{I}'}(\delta)$.

Since each entity, v, in an interpretation, I, determines exactly one extended Pollard feature structure, $\langle v, \mathsf{I}_v \rangle$, in the interpretation I, it follows with PROPOSITION 4 that each $\mathrm{Abst}_{\mathsf{I}}$ is functional:

Proposition 5. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, $\mathrm{Abst}_{\mathsf{I}}$ *is a total function from* U *to* $\mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$.

If v is an entity in an interpretation I, then I call $\mathrm{Abst}_{\mathsf{I}}(v)$ its *abstraction*. The strong generative capacity of a grammar is supposed to contain one isomorphic member to each utterance token in the natural language that is judged grammatical:

Definition 28. [SGC 1] *For each signature Σ,* SGC *is the total function from the set of grammars to $Pow(\mathbb{AFS}^{\Sigma}_{\text{eP}})$ such that, for each $\theta \subseteq \mathcal{D}^{\Sigma}$,*

$$\mathsf{SGC}(\langle \Sigma, \theta \rangle) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\text{eP}} \;\middle|\; \begin{array}{l} \textit{for some } \langle \Sigma, \theta \rangle \textit{ model} \\ \mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle, \\ \textit{for some } v \in \mathsf{U}, \\ \alpha = \mathrm{Abst}_{\mathsf{I}}(v) \end{array} \right\}.$$

For each grammar Γ, Pollard calls the set SGC(Γ) the *strong generative capacity of* Γ.

Exhaustive models provide an obvious alternative to obtaining the SGC of a grammar by taking the abstract feature structures that can be abstracted from the entities in all models. It is clearly sufficient to consider only one model if that model is big enough in the sense of containing congruent copies of all possible configurations under entities in all models, and to take the set of abstract feature structures that can be abstracted from the entities in such a model.

Following Pollard 1999, we can exploit the existence of a canonical extended Pollard feature structure for each entity in an interpretation for a new way of characterizing exhaustive models:

Definition 29. *For each signature Σ, for each $\theta \subseteq \mathcal{D}^{\Sigma}$,*

$\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$ *is an* **exhaustive** $\langle \Sigma, \theta \rangle$ **model** *iff*

I *is a $\langle \Sigma, \theta \rangle$ model, and for each $\langle \Sigma, \theta \rangle$ model $\mathsf{I}' = \langle \mathsf{U}', \mathsf{S}', \mathsf{A}', \mathsf{R}' \rangle$, for each $v' \in \mathsf{U}'$, there exists a $v \in \mathsf{U}$ such that $\mathrm{Abst}_{\mathsf{I}}(v) = \mathrm{Abst}_{\mathsf{I}'}(v')$.*

The function that is defined next maps each Σ interpretation I to the smallest set of extended Pollard abstract feature structures under Σ that, for each extended Pollard feature structure determined by some v in I, contains an element that is indiscernible from $\langle v, \mathsf{I}_v \rangle$:

Definition 30. *For each signature Σ,* ABST *is the large total function from the class of Σ interpretations to $Pow(\mathbb{AFS}^{\Sigma}_{\text{eP}})$ such that, for each Σ interpretation $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,*

$$\mathsf{ABST}(\mathsf{I}) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\text{eP}} \;\middle|\; \begin{array}{l} \textit{for some } v \in \mathsf{U}, \\ \alpha = \mathrm{Abst}_{\mathsf{I}}(v) \end{array} \right\}.$$

For each Σ interpretation I, I call ABST(I) the *set of extended Pollard abstract feature structures determined*

by I. The function ABST leads to the second definition of the strong generative capacity of a grammar:

Definition 31. [SGC 2] *For each signature Σ,* SGC *is the total function from the set of grammars to $Pow(\mathbb{AFS}^{\Sigma}_{\text{eP}})$ such that, for each $\theta \subseteq \mathcal{D}^{\Sigma}$,*

$$\mathsf{SGC}(\langle \Sigma, \theta \rangle) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\text{eP}} \;\middle|\; \begin{array}{l} \textit{for each (equivalently some)} \\ \textit{exhaustive } \langle \Sigma, \theta \rangle \textit{ model } \mathsf{I}, \\ \alpha \in \mathsf{ABST}(\mathsf{I}) \end{array} \right\}.$$

Finally, the third way of defining the strong generative capacity of a grammar exploits the fact that an extended Pollard abstract feature structure, in contrast to traditional definitions of abstract feature structures, contains an interpretation of a signature. In particular, it contains the interpretation with canonical representations of the elements of the universe that is generated by its root entity. Following Pollard, we say that a grammar Γ *generates* an extended Pollard abstract feature structure exactly if its interpretation is a Γ model:

Definition 32. *For each signature Σ, for each $\theta \subseteq \mathcal{D}^{\Sigma}$, for each Σ interpretation $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, for each $v \in \mathsf{U}$, $\langle \Sigma, \theta \rangle$* **generates** *$\langle v, \mathsf{I} \rangle$ iff*

$\langle v, \mathsf{I} \rangle \in \mathbb{AFS}^{\Sigma}_{\text{eP}}$*, and*

I *is a $\langle \Sigma, \theta \rangle$ model.*

This leads to the last characterization of the SGC of a grammar as the set of all extended Pollard abstract feature structures whose interpretation is a model of a grammar Γ:

Definition 33. [SGC 3] *For each signature Σ,* SGC *is the total function from the set of grammars to $Pow(\mathbb{AFS}^{\Sigma}_{\text{eP}})$ such that, for each $\theta \subseteq \mathcal{D}^{\Sigma}$,*

$$\mathsf{SGC}(\langle \Sigma, \theta \rangle) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\text{eP}} \;\middle|\; \langle \Sigma, \theta \rangle \textit{ generates } \alpha \right\}.$$

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