Introduction to Computational Linguistics

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What is in a State

Definition 4

Given a DFA M = $(\Sigma, Q, i, F, \delta)$,

a *state of M* is triple (x, q, y)

where $q \in Q$ and $x, y \in \Sigma^*$

The *directly derives* relation

Definition 5 (directly derives)

Given a DFA $(\Sigma, Q, i, F, \delta)$,

a state (x, q, y) *directly derives* state (x', q', y'):

$(x, q, y) \vdash (x', q', y')$ iff

1. there is $\sigma \in \Sigma$ such that y = σy' and x'= xσ (i.e. the reading head moves right one symbol σ)

2. $\delta(q, \sigma) = q'$

The *derives* relation

Definition 6 (derives)

Given a DFA $(\Sigma, Q, i, F, \delta)$,

a state A *derives* state B:

$(x, q, y) \vdash^{*} (x', q', y')$ iff

there is a sequence $S_0 \vdash S_1 \vdash \cdots \vdash S_k$

such that A = S_0 and B = S_k

Acceptance

Definition 7 (Acceptance)

Given a DFA $M = (\Sigma, Q, i, F, \delta)$ and a string $x \in \Sigma^*$,

M *accepts* x iff

there is a $q \in F$ such that $(0, i, x) \vdash {}^*(x, q, 0)$.

Language accepted by M

Definition 8 (Language accepted by M)

Given a DFA $M = (\Sigma, Q, i, F, \delta)$, the language $L(M)$ accepted by M is the set of all strings accepted by M.

Example of String Acceptance

Let M = $(\{a, b\}, \{q_0, q_1, q_2\}, q_0, \{q_1\}, \{((q_0, a), q_1), ((q_0, b), q_1), ((q_1, a), q_2), ((q_1, b), q_2), ((q_2, a), q_2), ((q_2, b), q_2), \})$.

Example of String Acceptance

Let M = $(\{a, b\}, \{q_0, q_1, q_2\}, q_0, \{q_1\}, \{((q_0, a), q_1), ((q_0, b), q_1),$
$((q_1, a), q_2), ((q_1, b), q_2), ((q_2, a), q_2), ((q_2, b), q_2), \})$.

M accepts a and b and nothing else, i.e. $L(M) = \{a, b\}$, since

$(0, q_0, a) \vdash (a, q_1, 0)$ and
$(0, q_0, b) \vdash (b, q_1, 0)$

are the only derivations from a start state to a final state for M.

More Properties of FSAs

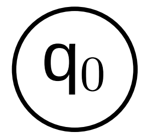
Given the FSAs A, A_1, and A_2 and the string w, the following properties are decidable:

Membership:	$w \stackrel{?}{\in} L(A)$
Emptiness:	$L(A) \stackrel{?}{=} \varnothing$
Totality:	$L(A) \stackrel{?}{=} \Sigma^*$
Subset:	$L(A_1) \stackrel{?}{\subseteq} L(A_2)$
Equality:	$L(A_1) \stackrel{?}{=} L(A_2)$

Regular Expressions and Automata (1)

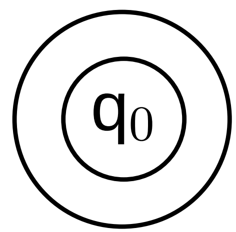
Regular Expression: Ø

Automaton:



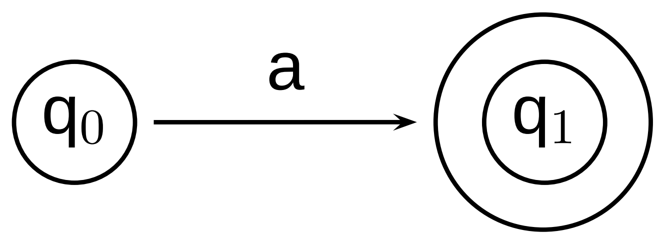
Regular Expression: O

Automaton:



Regular Expression: a

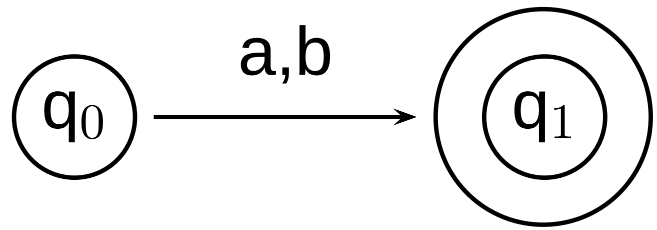
Automaton:



Regular Expressions and Automata (2)

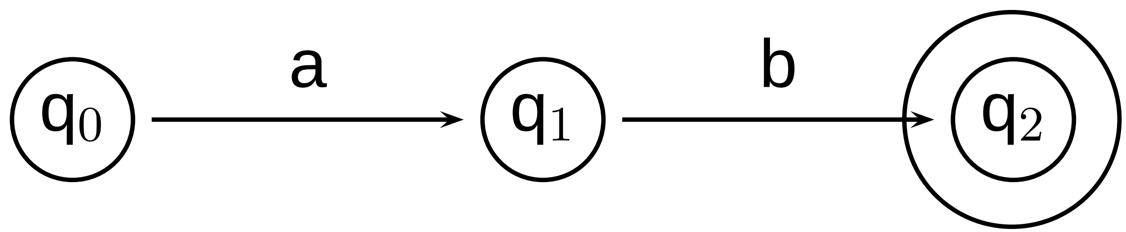
Regular Expression: [a | b]

Automaton:



Regular Expression: [a b]

Automaton:



The Finite State Utilities

The FSA Utilities toolbox:

- a collection of utilities to manipulate regular expressions, finite-state automata (and finite-state transducers).
- implemented in Prolog by Gertjan van Noord, University of Groningen
- Home Page:
  `http://odur.let.rug.nl/~vannoord/Fsa/`
- command in the SfS network (on ‘penthesilea’):
  `fsa -tk`

Reg. Expressions: Syntactic Extensions

$A	*contains*
	$A $=_{def}$ [?* A ?*]
	for example: $[a | b] denotes all strings
	that contain at least one a or b somewhere.
A & B	Intersection
A - B	Relative complement (minus)
$\sim$ A	Complement (negation)

The Bigger Picture

Definition 9 (Regular Languages)

A language L is said to be *regular or recognizable* if the set of strings s such that $s \in L$ are accepted by a DFA.

Theorem (Kleene, 1956)

The family of regular languages over Σ^* is equal to the smallest family of languages over Σ^* that contains the empty set, the singleton sets, and that is closed under Kleene star, concatenation, and union.

$\Rightarrow$ The family of regular languages over Σ^* is equal to the family of languages denoted by the set of regular expressions.