A Comparison of HPSG and LFG

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A Logical Language for HPSG

RSRL

The Description Language I

Some examples of traditional HPSG principles:

- The HEAD FEATURE PRINCIPLE
  In a headed phrase, the values of SYNSEM | LOCAL | CATEGORY | HEAD *and* DAUGHTERS | HEAD-DAUGHTER | SYNSEM | LOCAL | CATEGORY | HEAD *are token-identical.*

- The SUBJECT CONDITION (Pollard and Sag, 1994, p. 400)
  If the initial element of the SUBCAT *value of a word is slashed, then so is some other element of that list.*

- The RELATIVE UNIQUENESS PRINCIPLE (Pollard and Sag, 1994, p. 400)
  For any phrase, a member of the set value of SYNSEM | NONLOCAL | INHERITED | REL *may belong to the value of that same path on at most one daughter.*

- Binding Theory, Principle B (Pollard and Sag, 1994, p. 401)
  A personal pronoun must be locally o-free.

A Principle from Abeillé and Godard 1996:

In the original notation:

$$\begin{bmatrix}\text{VFORM} & \textit{part-passé}\\ \text{S-ARG} & \langle \ldots \textit{aff\_r} \ldots \rangle\end{bmatrix} \rightarrow [\text{V-AUX} \ \ \textit{etre}]$$

A formalization in RSRL:

$$\left(\begin{array}{l}\exists\boxed{0}\,\exists\boxed{1}\\ \left(\begin{array}{l}\begin{bmatrix}\text{VFORM} & \textit{part-passé}\\ \text{S-ARG} & \boxed{1}\end{bmatrix}\\ \wedge\, \texttt{member}\,(\boxed{0}[\textit{aff\_r}], \boxed{1})\end{array}\right)\end{array}\right) \rightarrow [\text{V-AUX} \ \ \textit{etre}]$$

Signatures

Definition 1. Σ *is a* **signature** *iff*

Σ is a septuple $\langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$\langle \mathcal{G}, \sqsubseteq \rangle$ is a partial order,

$$\mathcal{S} = \left\{ \sigma \in \mathcal{G} \,\middle|\, \begin{array}{l} \text{for each } \sigma' \in \mathcal{G}, \\ \text{if } \sigma' \sqsubseteq \sigma \text{ then } \sigma = \sigma' \end{array} \right\},$$

$\mathcal{A}$ is a set,

$\mathcal{F}$ is a partial function from the Cartesian product of $\mathcal{G}$ and $\mathcal{A}$ to $\mathcal{G}$,

for each $\sigma_1 \in \mathcal{G}$, for each $\sigma_2 \in \mathcal{G}$ and for each $\alpha \in \mathcal{A}$,

if $\mathcal{F}\langle \sigma_1, \alpha \rangle$ is defined and $\sigma_2 \sqsubseteq \sigma_1$
then $\mathcal{F}\langle \sigma_2, \alpha \rangle$ is defined and $\mathcal{F}\langle \sigma_2, \alpha \rangle \sqsubseteq \mathcal{F}\langle \sigma_1, \alpha \rangle$,

$\mathcal{R}$ is a finite set, and

$\mathcal{AR}$ is a total function from $\mathcal{R}$ to $\mathbb{N}^+$.

Interpretations

Definition 2. *For each signature $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, I is a Σ* **interpretation** *iff*

I is a quadruple $\langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

U is a set,

S is a total function from U to $\mathcal{S}$,

A is a total function from $\mathcal{A}$ to the set of partial functions from U to U,

for each $\alpha \in \mathcal{A}$ and each $u \in \mathsf{U}$,

if $\mathsf{A}(\alpha)(u)$ is defined
then $\mathcal{F}\langle \mathsf{S}(u), \alpha \rangle$ is defined, and $\mathsf{S}(\mathsf{A}(\alpha)(u)) \sqsubseteq \mathcal{F}\langle \mathsf{S}(u), \alpha \rangle$, and

for each $\alpha \in \mathcal{A}$ and each $u \in \mathsf{U}$,

if $\mathcal{F}\langle \mathsf{S}(u), \alpha \rangle$ is defined then $\mathsf{A}(\alpha)(u)$ is defined,

R is a total function from $\mathcal{R}$ to the power set of $\bigcup_{n \in \mathbb{N}} \overline{\mathsf{U}}^n$, and

for each $\rho \in \mathcal{R}$, $\mathsf{R}(\rho) \subseteq \overline{\mathsf{U}}^{\mathcal{AR}(\rho)}$.

Chains

Definition 3. *For each signature $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,*

$\widehat{\mathcal{G}} = \mathcal{G} \cup \{chain, echain, nechain, metatop\}$,

$\widehat{\sqsubseteq} = \sqsubseteq \cup \{\langle echain, chain \rangle, \langle nechain, chain \rangle\} \cup \left\{ \langle \sigma, \sigma \rangle \,\middle|\, \sigma \in \widehat{\mathcal{G}} \backslash \mathcal{G} \right\} \cup \left\{ \langle \sigma, metatop \rangle \,\middle|\, \sigma \in \widehat{\mathcal{G}} \right\}$,

$\widehat{\mathcal{S}} = \mathcal{S} \cup \{echain, nechain\}$, *and*

$\widehat{\mathcal{A}} = \mathcal{A} \cup \{\dagger, \triangleright\}$.

Definition 4. *For each signature $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, for each Σ interpretation $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,*

$\widehat{\mathsf{S}}$ is the total function from $\overline{\mathsf{U}}$ to $\widehat{\mathcal{S}}$ such that

for each $u \in \mathsf{U}$, $\widehat{\mathsf{S}}(u) = \mathsf{S}(u)$,
for each $u_1 \in \mathsf{U}, \ldots,$ for each $u_n \in \mathsf{U}$,

$$\widehat{\mathsf{S}}(\langle u_1, \ldots, u_n \rangle) = \begin{cases} echain & \text{if } n = 0, \\ nechain & \text{if } n > 0 \end{cases}, \text{ and}$$

$\widehat{\mathsf{A}}$ is the total function from $\widehat{\mathcal{A}}$ to the set of partial functions from $\overline{\mathsf{U}}$ to $\overline{\mathsf{U}}$ such that

for each $\alpha \in \mathcal{A}$, $\widehat{\mathsf{A}}(\alpha) = \mathsf{A}(\alpha)$,
$\widehat{\mathsf{A}}(\dagger)$ is the total function from U^+ to U such that for each $\langle u_0, \ldots, u_n \rangle \in \mathsf{U}^+$,
$\widehat{\mathsf{A}}(\dagger)(\langle u_0, \ldots, u_n \rangle) = u_0$, and
$\widehat{\mathsf{A}}(\triangleright)$ is the total function from U^+ to U^* such that for each $\langle u_0, \ldots, u_n \rangle \in \mathsf{U}^+$,
$\widehat{\mathsf{A}}(\triangleright)(\langle u_0, \ldots, u_n \rangle) = \langle u_1, \ldots, u_n \rangle$.

Formulae

Definition 5. *For each signature* Σ, $\mathcal{T}^\Sigma$ *and* $\mathcal{D}^\Sigma$ *are the smallest sets such that*

$: \in \mathcal{T}^\Sigma$,

for each $v \in \mathcal{VAR}$, $v \in \mathcal{T}^\Sigma$,

for each $\alpha \in \widehat{\mathcal{A}}$ *and each* $\tau \in \mathcal{T}^\Sigma$, $\tau\alpha \in \mathcal{T}^\Sigma$,

for each $\sigma \in \widehat{\mathcal{G}}$, *for each* $\tau \in \mathcal{T}^\Sigma$, $\tau{\sim}\sigma \in \mathcal{D}^\Sigma$,

for each $\tau_1 \in \mathcal{T}^\Sigma$, *for each* $\tau_2 \in \mathcal{T}^\Sigma$, $\tau_1{\approx}\tau_2 \in \mathcal{D}^\Sigma$,

for each $\rho \in \mathcal{R}$, *for each* $x_1 \in \mathcal{VAR}$, ..., *for each* $x_{\mathcal{AR}(\rho)} \in \mathcal{VAR}$,

$\rho(x_1, \ldots, x_{\mathcal{AR}(\rho)}) \in \mathcal{D}^\Sigma$,

for each $x \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$, $\exists x\, \delta \in \mathcal{D}^\Sigma$,

for each $x \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$, $\forall x\, \delta \in \mathcal{D}^\Sigma$,

for each $\delta \in \mathcal{D}^\Sigma$, $\neg\delta \in \mathcal{D}^\Sigma$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$, $[\delta_1 \wedge \delta_2] \in \mathcal{D}^\Sigma$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$, $[\delta_1 \vee \delta_2] \in \mathcal{D}^\Sigma$.

The Description Language II

- The TRACE PRINCIPLE (Pollard and Sag, 1994, p. 400)
  The SYNSEM *value of any trace must be a (noninitial) member of the* SUBCAT *list of a substantive word.*

- The CONTROL THEORY (Pollard and Sag, 1994, p. 401)
  (looks for potential controlled *local* objects)
  If the SOA-ARG *value of a control-qfpsoa is token-identical with the* CONTENT *value of a local entity whose* CATEGORY | SUBCAT *value is a list of length one, then the member of that list is (1) reflexive, and (2) coindexed with the* INFLUENCED *(respectively,* COMMITTOR, EXPERIENCER*) value of the control-qfpsoa if the latter is of sort influence (respectively, commitment, orientation).*

- BINDING THEORY
  (looks for potential o-commanders)

- QUANTIFIER BINDING CONDITION (Pollard and Sag, 1994, p. 327)

Given a quantifier contained within a CONTENT *value, every occurrence within that* CONTENT *value of the quantifier's index must be captured by the quantifier.*

Formula Denotation

Definition 6. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

$Ass_{\mathsf{I}} = \overline{\mathsf{U}}^{\mathcal{VAR}}$ *is the* ***set of variable assignments in*** I.

Definition 7. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $ass \in Ass_{\mathsf{I}}$, T^{ass}_{I} *is the total function from* $\mathcal{T}^\Sigma$ *to the set of partial functions from* U *to* $\overline{\mathsf{U}}$ *such that for each* $u \in \mathsf{U}$,

$T^{ass}_{\mathsf{I}}(:)(u)$ *is defined and* $T^{ass}_{\mathsf{I}}(:)(u) = u$,

for each $v \in \mathcal{VAR}$, $T^{ass}_{\mathsf{I}}(v)(u)$ *is defined and* $T^{ass}_{\mathsf{I}}(v)(u) = ass(v)$,

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\alpha \in \widehat{\mathcal{A}}$,

$T^{ass}_{\mathsf{I}}(\tau\alpha)(u)$ *is defined*
iff $T^{ass}_{\mathsf{I}}(\tau)(u)$ *is defined and* $\widehat{\mathsf{A}}(\alpha)(T^{ass}_{\mathsf{I}}(\tau)(u))$ *is defined, and*
if $T^{ass}_{\mathsf{I}}(\tau\alpha)(u)$ *is defined then* $T^{ass}_{\mathsf{I}}(\tau\alpha)(u) = \widehat{\mathsf{A}}(\alpha)(T^{ass}_{\mathsf{I}}(\tau)(u))$.

Definition 8. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *and for each* $u \in \mathsf{U}$,

$$\mathsf{Co}^u_\mathsf{I} = \left\{ u' \in \mathsf{U} \,\middle|\, \begin{array}{l} \text{for some } ass \in Ass_\mathsf{I}, \\ \text{for some } \pi \in \mathcal{A}^*, \\ \quad T^{ass}_\mathsf{I}(:\pi)(u) \text{ is defined, and} \\ \quad u' = T^{ass}_\mathsf{I}(:\pi)(u) \end{array} \right\}.$$

Definition 9. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $ass \in Ass_\mathsf{I}$, *for each* $v \in \mathcal{VAR}$, *for each* $w \in \mathcal{VAR}$, *for each* $u \in \overline{\mathsf{U}}$,

$$ass\tfrac{u}{v}(w) = \begin{cases} u & \text{if } v = w \\ ass(w) & \text{otherwise.} \end{cases}$$

Definition 10. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $ass \in Ass_\mathsf{I}$, D^{ass}_I *is the total function from* $\mathcal{D}^\Sigma$ *to the power set of* U *such that*

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\sigma \in \widehat{\mathcal{G}}$,

$$D^{ass}_\mathsf{I}(\tau \sim \sigma) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} T^{ass}_\mathsf{I}(\tau)(u) \text{ is defined, and} \\ \widehat{\mathsf{S}}(T^{ass}_\mathsf{I}(\tau)(u)) \mathrel{\widehat{\sqsubseteq}} \sigma \end{array} \right\},$$

for each $\tau_1 \in \mathcal{T}^\Sigma$, *for each* $\tau_2 \in \mathcal{T}^\Sigma$,

$$D^{ass}_\mathsf{I}(\tau_1 \approx \tau_2) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} T^{ass}_\mathsf{I}(\tau_1)(u) \text{ is defined,} \\ T^{ass}_\mathsf{I}(\tau_2)(u) \text{ is defined, and} \\ T^{ass}_\mathsf{I}(\tau_1)(u) = T^{ass}_\mathsf{I}(\tau_2)(u) \end{array} \right\},$$

for each $\rho \in \mathcal{R}$, *for each* $x_1 \in \mathcal{VAR}$, ..., *for each* $x_{\mathcal{AR}(\rho)} \in \mathcal{VAR}$,

$$D^{ass}_\mathsf{I}(\rho(x_1, \ldots, x_{\mathcal{AR}(\rho)})) = \left\{ u \in \mathsf{U} \,\middle|\, \langle ass(x_1), \ldots, ass(x_{\mathcal{AR}(\rho)}) \rangle \in \mathsf{R}(\rho) \right\},$$

for each $v \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$,

$$D^{ass}_\mathsf{I}(\exists v\ \delta) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} \text{for some } u' \in \overline{\mathsf{Co}^u_\mathsf{I}}, \\ u \in D^{ass\frac{u'}{v}}_\mathsf{I}(\delta) \end{array} \right\},$$

for each $v \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$,

$$D^{ass}_\mathsf{I}(\forall v\ \delta) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} \text{for each } u' \in \overline{\mathsf{Co}^u_\mathsf{I}}, \\ u \in D^{ass\frac{u'}{v}}_\mathsf{I}(\delta) \end{array} \right\},$$

for each $\delta \in \mathcal{D}^\Sigma$,

$$D^{ass}_\mathsf{I}(\neg\delta) = \mathsf{U} \backslash D^{ass}_\mathsf{I}(\delta),$$

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$D^{ass}_\mathsf{I}([\delta_1 \wedge \delta_2]) = D^{ass}_\mathsf{I}(\delta_1) \cap D^{ass}_\mathsf{I}(\delta_2),$$

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$D^{ass}_\mathsf{I}([\delta_1 \vee \delta_2]) = D^{ass}_\mathsf{I}(\delta_1) \cup D^{ass}_\mathsf{I}(\delta_2)$, *and*

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$D^{ass}_\mathsf{I}([\delta_1 \rightarrow \delta_2]) = (\mathsf{U} \backslash D^{ass}_\mathsf{I}(\delta)) \cup D^{ass}_\mathsf{I}(\delta_2).$$

From Formulae to Descriptions

Definition 11. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$FV(:) = \emptyset$,

for each $v \in \mathcal{VAR}$, $FV(v) = \{v\}$,

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\alpha \in \widehat{\mathcal{A}}$, $FV(\tau\alpha) = FV(\tau)$,

for each $\tau \in \mathcal{T}^\Sigma$, *for each* $\sigma \in \widehat{\mathcal{G}}$, $FV(\tau \sim \sigma) = FV(\tau)$,

for each $\tau_1, \tau_2 \in \mathcal{T}^\Sigma$, $FV(\tau_1 \approx \tau_2) = FV(\tau_1) \cup FV(\tau_2)$,

for each $\rho \in \mathcal{R}$, *for each* $x_1, \ldots, x_{\mathcal{AR}(\rho)} \in \mathcal{VAR}$,

$FV(\rho(x_1, \ldots, x_{\mathcal{AR}(\rho)})) = \{x_1, \ldots, x_{\mathcal{AR}(\rho)}\}$,

for each $\delta \in \mathcal{D}^\Sigma$, *for each* $v \in \mathcal{VAR}$, $FV(\exists v\ \delta) = FV(\delta)\backslash\{v\}$,

for each $\delta \in \mathcal{D}^\Sigma$, *for each* $v \in \mathcal{VAR}$, $FV(\forall v\ \delta) = FV(\delta)\backslash\{v\}$,

for each $\delta \in \mathcal{D}^\Sigma$, $FV(\neg\delta) = FV(\delta)$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$FV([\delta_1 \wedge \delta_2]) = FV(\delta_1) \cup FV(\delta_2)$,

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$FV([\delta_1 \vee \delta_2]) = FV(\delta_1) \cup FV(\delta_2)$, *and*

for each $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$FV([\delta_1 \rightarrow \delta_2]) = FV(\delta_1) \cup FV(\delta_2)$.

Proposition 1. *For each signature* Σ, *for each* Σ *interpretation* I, *for each* $ass_1 \in Ass_\mathsf{I}$, *for each* $ass_2 \in Ass_\mathsf{I}$,

for each $\tau \in \mathcal{T}^\Sigma$,

if for each $v \in FV(\tau)$, $ass_1(v) = ass_2(v)$ *then* $T_\mathsf{I}^{ass_1}(\tau) = T_\mathsf{I}^{ass_2}(\tau)$, *and*

for each $\delta \in \mathcal{D}^\Sigma$,

if for each $v \in FV(\delta)$, $ass_1(v) = ass_2(v)$ *then* $D_\mathsf{I}^{ass_1}(\delta) = D_\mathsf{I}^{ass_2}(\delta)$.

Definition 12. *For each signature* Σ,

$$\mathcal{D}_0^\Sigma = \{\delta \in \mathcal{D}^\Sigma \mid FV(\delta) = \emptyset\}.$$

Corollary 1. *For each signature* Σ, *for each* $\delta \in \mathcal{D}_0^\Sigma$, *for each* Σ *interpretation* I, *for each* $ass_1 \in Ass_\mathsf{I}$, *for each* $ass_2 \in Ass_\mathsf{I}$,

$$D_\mathsf{I}^{ass_1}(\delta) = D_\mathsf{I}^{ass_2}(\delta).$$

Description Denotation and Theory Denotation

Definition 13. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, D_I *is the total function from* $\mathcal{D}_0^\Sigma$ *to the power set of* U *such that for each* $\delta \in \mathcal{D}_0^\Sigma$,

$$D_\mathsf{I}(\delta) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} \text{for each } ass \in Ass_\mathsf{I}, \\ u \in D_\mathsf{I}^{ass}(\delta) \end{array} \right\}.$$

Definition 14. *For each signature* Σ, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, Θ_I *is the total function from the power set of* $\mathcal{D}_0^\Sigma$ *to the power set of* U *such that for each* $\theta \subseteq \mathcal{D}_0^\Sigma$,

$$\Theta_\mathsf{I}(\theta) = \left\{ u \in \mathsf{U} \,\middle|\, \begin{array}{l} \text{for each } \delta \in \theta, \\ u \in D_\mathsf{I}(\delta) \end{array} \right\}.$$

Grammars and Models

Definition 15. Γ *is a* ***grammar*** *iff*

Γ *is a pair* $\langle \Sigma, \theta \rangle$,

Σ *is a signature, and*

$\theta \subseteq \mathcal{D}_0^\Sigma$.

Definition 16. *For each grammar* $\Gamma = \langle \Sigma, \theta \rangle$, *for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

I *is a* Γ ***model*** *iff* $\Theta_\mathsf{I}(\theta) = \mathsf{U}$.

King 1999 in RSRL

An Example

A Signature:

top

 car OWNER $person$
  DRIVER $person$

  vw

  bmw

 $person$ LIKES-BEST top

  man

  $woman$

$\mathcal{R} = \{\}$, $\mathcal{AR} = \{\}$

An interpretation of our signature:
Let $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, with:

$\mathsf{U} = \left\{ \begin{array}{l} \text{Anke, Anke's husband, Detmar, Kordula,} \\ \text{first car, second car} \end{array} \right\}$

S(Anke) $= woman$,
S(Kordula) $= woman$,
S(Anke's husband) $= man$,
S(Detmar) $= man$,
S(first car) $= vw$, and
S(second car) $= vw$
A(LIKES-BEST)(Anke) = Anke's husband,
A(LIKES-BEST)(Anke's husband) = Anke,
A(LIKES-BEST)(Detmar) = Kordula,
A(LIKES-BEST)(Kordula) = Detmar,
A(OWNER)(first car) = Kordula,
A(OWNER)(second car) = Anke,
A(DRIVER)(first car) = Detmar,
A(DRIVER)(second car) = Anke,
A(LIKES-BEST) is undefined on the first car and on the second car,
A(OWNER) is undefined on Anke, Anke's husband, Detmar and Kordula, and
A(DRIVER) is likewise undefined on Anke, Anke's husband, Detmar and Kordula, and
$\mathsf{R} = \{\}$.

Grammars of I:

Assume $\theta_1 = \left\{ \begin{array}{l} [: \sim bmw \rightarrow \neg : \approx :], \\ [: \sim vw \rightarrow : \text{OWNER} \sim woman] \end{array} \right\}$.

Observation:
I is a model of $\langle \Sigma, \theta_1 \rangle$. However, $\langle \Sigma, \theta_1 \rangle$ overlicenses I.
I_2 is a relevant example:

Let $\mathsf{I}_2 = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, with:
U = {Anne, Manfred, Maria, Mike, third car}
S(Anne) $= woman$,
S(Maria) $= woman$,
S(Manfred) $= man$,
S(Mike) $= man$,
S(third car) $= vw$, and
A(LIKES-BEST)(Anne) = Manfred,
A(LIKES-BEST)(Manfred) = Anne,
A(LIKES-BEST)(Maria) = Mike,
A(LIKES-BEST)(Mike) = Maria,
A(OWNER)(third car) = Anne,
A(DRIVER)(third car) = Mike,
A(LIKES-BEST) is undefined on the third car,
A(OWNER) is undefined on Anne, Manfred, Maria and Mike, and

$\mathsf{A}(\textsc{driver})$ is likewise undefined on Anne, Manfred, Maria and Mike, and
$\mathsf{R} = \{\}$.

Now assume

$$\theta_2 = \left\{ \begin{array}{l} [: \sim woman \leftrightarrow [: \textsc{likes-best} \sim man \land : \textsc{likes-best likes-best} \approx:]] , \\ [: \sim man \leftrightarrow [: \textsc{likes-best} \sim woman \land : \textsc{likes-best likes-best} \approx:]] , \\ [: \sim bmw \to \neg : \approx:] , \\ [: \sim vw \to [: \textsc{owner} \sim woman \land : \textsc{owner} \approx: \textsc{driver}]] \end{array} \right\}.$$

Problem:
$\langle \Sigma, \theta_2 \rangle$ underlicenses I. I is not even a model of $\langle \Sigma, \theta_2 \rangle$.

Finally, consider

$$\theta_3 = \left\{ \begin{array}{l} [: \sim woman \leftrightarrow [: \textsc{likes-best} \sim man \land : \textsc{likes-best likes-best} \approx:]] , \\ [: \sim man \leftrightarrow [: \textsc{likes-best} \sim woman \land : \textsc{likes-best likes-best} \approx:]] , \\ [: \sim bmw \to \neg : \approx:] , \\ \left[: \sim vw \to \left[\begin{array}{l} : \textsc{owner} \sim woman \land \\ [: \textsc{owner} \approx: \textsc{driver} \lor : \textsc{driver} \approx: \textsc{owner likes-best}] \end{array} \right] \right] \end{array} \right\}.$$

Observation:
I is an exhaustive model of $\langle \Sigma, \theta_3 \rangle$.

Terminology

Definition 17. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$*, for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$*, for each* $u \in \mathsf{I}$*,*

> $\mathsf{I}_u = \langle \mathsf{U}_u, \mathsf{S}_u, \mathsf{A}_u, \mathsf{R}_u \rangle$ *is the* Σ ***interpretation under*** u ***in*** I *iff*
>
> $\mathsf{U}_u = \left\{ u' \in \mathsf{U} \,\middle|\, u' \in \mathsf{Co}_{\mathsf{I}}^{u} \right\}$,
> $\mathsf{S}_u = \mathsf{S} \cap (\mathsf{U}_u \times \mathcal{S})$,
> $\mathsf{A}_u = \mathsf{A} \cap (\mathcal{A} \times \{\mathsf{U}_u \times \mathsf{U}_u\})$,
> $\mathsf{R}_u = \mathsf{R} \cap \left(\mathcal{R} \times Pow \left(\bigcup_{n \in \mathbb{N}} \left(\overline{\mathsf{U}}_u\right)^n \right) \right)$*, and*
> $\mathsf{I}_u = \langle \mathsf{U}_u, \mathsf{S}_u, \mathsf{A}_u, \mathsf{R}_u \rangle$.

Definition 18. *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$*, for each* Σ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$*, for each* $u \in \mathsf{I}$*,*

> $\langle u, \mathsf{I}_u \rangle$ *is the* ***configuration of entities under*** u ***in*** I *iff*
>
> I_u *is the* Σ *interpretation under* u *in* I.

Definition 19. *For each signature* Σ*, for each* Σ *interpretation* $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1 \rangle$*, for each* $u_1 \in \mathsf{U}_1$*, for each* Σ *interpretation* $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2 \rangle$*, for each* $u_2 \in \mathsf{U}_2$*,*

> f *is a* ***congruence*** *from* $\langle u_1, \mathsf{I}_1 \rangle$ *to* $\langle u_2, \mathsf{I}_2 \rangle$ *in* Σ *iff*
>
> f *is a bijection from* $\overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$ *to* $\overline{\mathsf{Co}_{\mathsf{I}_2}^{u_2}}$*,*
> *for each* $u \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$*,* $\widehat{\mathsf{S}_1}(u) = \widehat{\mathsf{S}_2}(f(u))$*,*
> *for each* $\alpha \in \widehat{\mathcal{A}}$*, for each* $u \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$*,*
> $\widehat{\mathsf{A}_1}(\alpha)(u)$ *is defined iff* $\widehat{\mathsf{A}_2}(\alpha)(f(u))$ *is defined, and*
> *if* $\widehat{\mathsf{A}_1}(\alpha)(u)$ *is defined then* $f(\widehat{\mathsf{A}_1}(\alpha)(u)) = \widehat{\mathsf{A}_2}(\alpha)(f(u))$*,*
> *for each* $\rho \in \mathcal{R}$*, for each* $o_1 \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$*, . . . , for each* $o_{\mathcal{AR}(\rho)} \in \overline{\mathsf{Co}_{\mathsf{I}_1}^{u_1}}$*,*
> $\langle o_1, \ldots, o_{\mathcal{AR}(\rho)} \rangle \in \mathsf{R}_1(\rho)$
> *iff* $\langle f(o_1), \ldots, f(o_{\mathcal{AR}(\rho)}) \rangle \in \mathsf{R}_2(\rho)$*, and*
> $f(u_1) = u_2$.

Definition 20. *For each signature* Σ*, for each* Σ *interpretation* $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1 \rangle$*, for each* $u_1 \in \mathsf{U}_1$*, for each* Σ *interpretation* $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2 \rangle$*, for each* $u_2 \in \mathsf{U}_2$*,*

> $\langle u_1, \mathsf{I}_1 \rangle$ *and* $\langle u_2, \mathsf{I}_2 \rangle$ *are* ***congruent in*** Σ
> *iff for some* f*,* f *is a congruence from* $\langle u_1, \mathsf{I}_1 \rangle$ *to* $\langle u_2, \mathsf{I}_2 \rangle$ *in* Σ.

Linguistic Truth in King 1999

The three conditions of King 1999, p. 343, can now straightforwardly be cast in terms of RSRL. The only difference to the original three conditions is that I am claiming that natural languages include relations between components (and chains of components) of tokens. A grammar $\langle \Sigma, \theta \rangle$ is true of a natural language only if

1. the natural language can be construed as an interpretation, I, of the signature Σ, where I is a system of linguistic tokens,

2. each description in θ is true of each entity in the natural language, i.e., the natural language, I, is a $\langle \Sigma, \theta \rangle$ model, and

3. some description in θ is false of some component of each entity, u', in any Σ interpretation, I', for which no entity u in the natural language, I, has isomorphically configured components.

Except for the reference to the extended definitions of all relevant terms, the conditions 1–3 are identical to

King's conditions on SRL grammars. The third condition amounts to saying that the language that is described by a grammar is in the class of exhaustive models of the grammar. Since we know that for each grammar, there exists an exhaustive model, it follows that King's interpretation of the meaning of a grammar is fully compatible with RSRL.

Exhaustive Models

Based on the notion of congruence, we can give an algebraic characterization of exhaustive models:

Definition 21. *For each signature Σ, for each Σ interpretation $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1 \rangle$, for each Σ interpretation $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2 \rangle$,*

*I_1 **simulates** I_2 **in** Σ iff
for each $u_2 \in \mathsf{U}_2$, for some $u_1 \in \mathsf{U}_1$, $\langle u_1, \mathsf{I}_1 \rangle$ and $\langle u_2, \mathsf{I}_2 \rangle$ are congruent in Σ.*

Definition 22. *For each signature Σ, for each $\theta \subseteq \mathcal{D}_0^\Sigma$, for each Σ interpretation I,*

*I is an **exhaustive** $\langle \Sigma, \theta \rangle$ **model** iff*

*I is a $\langle \Sigma, \theta \rangle$ model, and
for each Σ interpretation I',
if I' is a $\langle \Sigma, \theta \rangle$ model then I simulates I' in Σ.*

A simpler characterization of exhaustive models can be based on description denotations alone:

Definition 23. *For each signature Σ, for each Σ interpretation $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1 \rangle$, for each $u_1 \in \mathsf{U}_1$, for each Σ interpretation $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2 \rangle$, for each $u_2 \in \mathsf{U}_2$,*

*$\langle u_1, \mathsf{I}_1 \rangle$ and $\langle u_2, \mathsf{I}_2 \rangle$ are **indiscernible in** Σ
iff for each $\delta \in \mathcal{D}_0^\Sigma$, $u_1 \in D_{\mathsf{I}_1}(\delta)$ iff $u_2 \in D_{\mathsf{I}_2}(\delta)$.*

Indiscernibility and congruence turn out to be equivalent:

Proposition 2. *For each signature Σ, for each Σ interpretation $\mathsf{I}_1 = \langle \mathsf{U}_1, \mathsf{S}_1, \mathsf{A}_1, \mathsf{R}_1 \rangle$, for each $o_1 \in \mathsf{U}_1$, for each Σ interpretation $\mathsf{I}_2 = \langle \mathsf{U}_2, \mathsf{S}_2, \mathsf{A}_2, \mathsf{R}_2 \rangle$, for each $o_2 \in \mathsf{U}_2$,*

$\langle o_1, \mathsf{I}_1 \rangle$ and $\langle o_2, \mathsf{I}_2 \rangle$ are congruent in Σ iff $\langle o_1, \mathsf{I}_1 \rangle$ and $\langle o_2, \mathsf{I}_2 \rangle$ are indiscernible in Σ.

Since indiscernibility and congruence are equivalent, the simulation of models of grammars can alternatively be expressed in terms of the denotation of sets of descriptions in interpretations of the same signature.

One Σ model I of a grammar, $\langle\Sigma,\theta\rangle$, simulates another Σ model I$'$ of the grammar in Σ only if each Σ theory that describes something in I$'$ also describes something in I.

Proposition 3. *For each signature Σ, for each $\theta \subseteq \mathcal{D}_0^\Sigma$, for each Σ interpretation* I*,*

for each Σ interpretation I$'$*,*

if I$'$ *is a $\langle\Sigma,\theta\rangle$ model then* I *simulates* I$'$ *in Σ*

iff for each $\theta' \subseteq \mathcal{D}_0^\Sigma$, for each Σ interpretation I$'$*,*

if I$'$ *is a $\langle\Sigma,\theta\rangle$ model and $\Theta_{\mathsf{I}'}(\theta') \neq \emptyset$ then $\Theta_{\mathsf{I}}(\theta') \neq \emptyset$.*

Corollary 2. *For each grammar $\Gamma = \langle\Sigma,\theta\rangle$, for each Σ interpretation* I*,*

I *is an exhaustive Γ model iff*

I *is a Γ model, and*
for each $\theta' \subseteq \mathcal{D}_0^\Sigma$, for each Σ interpretation I$'$*,*
if I$'$ *is a Γ model and $\Theta_{\mathsf{I}'}(\theta') \neq \emptyset$ then $\Theta_{\mathsf{I}}(\theta') \neq \emptyset$.*

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