# A Comparison of HPSG and LFG

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## The Exhaustive Models Theorem of RSRL

**Theorem 1.** *For each signature $\Sigma$, for each $\Sigma$ theory $\theta$, there exists a $\Sigma$ interpretation* I *such that* I *is an exhaustive $\langle \Sigma, \theta \rangle$ model.*

As a corollary of THEOREM 1, we get immediately that if an RSRL grammar has a nonempty model, it also has a nonempty exhaustive model. In summation, a non-vacuous grammar has a linguistically sensible meaning in the class of its exhaustive models. King's explanation of the meaning of grammars, *mutatis mutandis*, carries over to RSRL.

A proof of THEOREM 1, due to Richter and King 1997, can be found in Appendix A of Richter 2000.

# Pollard 1999 in RSRL



## Pollard Feature Structures

On the previous handout, page 26, we introduced the notions of a "$\Sigma$ interpretation under $u$ in I, $\mathsf{I}_u$" and of a "configuration of entities under $u$ in I, $\langle u, \mathsf{I}_u \rangle$". For each signature $\Sigma$, for each $\Sigma$ interpretation I, and for each $u \in$ I, Pollard 1999 calls the counterpart in SRL of $\mathsf{I}_u$ a $\Sigma$ *interpretation generated by $u$ in* I; and, for each signature $\Sigma$, for each $\Sigma$ interpretation I, and for each $u \in$ I, Pollard 1999 calls the SRL counterpart of $\langle u, \mathsf{I}_u \rangle$ a *feature structure* (determined by $u$ in I). Since these mathematical structures differ from standard feature structures in important ways, and since we are concerned with a structural extension of Pollard's entities, I call $\langle u, \mathsf{I}_u \rangle$ *extended Pollard feature structures*.

According to their definition, the extended Pollard feature structures are Pollard's analogue to King's linguistic tokens. To obtain his unique representational idealizations of isomorphic utterance tokens, Pollard must provide a canonical representation of isomorphic utterance tokens. It is the purpose of the following definition of a *node abstraction* to provide the necessary prerequisites:

**Definition 1.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, $\nu$ *is a* **node abstraction in** $\Sigma$ *iff*

$\nu$ *is a pair* $\langle \beta, \varrho \rangle$,

$\beta \subseteq \mathcal{T}^{\Sigma}$,

$: \in \beta$,

*for each* $\tau \in \mathcal{T}^{\Sigma}$, *for each* $\varphi \in \mathcal{A}$, *if* $\tau\varphi \in \beta$ *then* $\tau \in \beta$,

$\varrho$ *is an equivalence relation over* $\beta$,

*for each* $\tau_1 \in \mathcal{T}^{\Sigma}$, *for each* $\tau_2 \in \mathcal{T}^{\Sigma}$, *for each* $\varphi \in \mathcal{A}$,

*if* $\tau_1\varphi \in \beta$ *and* $\langle \tau_1, \tau_2 \rangle \in \varrho$ *then* $\langle \tau_1\varphi, \tau_2\varphi \rangle \in \varrho$.

$\beta$ is a prefix closed set of terms that contains at least the colon, and $\varrho$ is a right invariant equivalence relation over $\beta$.

*Extended Pollard abstract feature structures* are canonical representations of extended Pollard feature structures. Their nodes are equivalence classes of terms in $\beta$, and their root entities are in the equivalence class $|:|_{\varrho}$:



**Definition 2.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* $\Sigma$ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $u \in \mathsf{U}$,

$\langle u, \mathsf{I} \rangle$ *is an* **extended Pollard abstract feature structure under** $\Sigma$ *iff*

$\langle u, \mathsf{I} \rangle$ *is an extended Pollard feature structure,*
*for some node abstraction in* $\Sigma$, $\langle \beta, \varrho \rangle$,
$\mathsf{U} = Quo(\langle \beta, \varrho \rangle)$,
$u = |:|_{\varrho}$,
*for each* $u_1 \in Quo(\langle \beta, \varrho \rangle)$, *for each* $u_2 \in Quo(\langle \beta, \varrho \rangle)$, *for each* $\varphi \in \mathcal{A}$,
$\mathsf{A}(\varphi)(u_1) = u_2$ *iff*
*for some* $\tau \in \beta$, $u_1 = |\tau|_{\varrho}$, *and* $u_2 = |\tau\varphi|_{\varrho}$.

I write $\mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$ for the set of extended Pollard abstract feature structures under $\Sigma$. We obtain the following crucial result:

**Proposition 1.** *For each signature* $\Sigma$, *for each* $\Sigma$ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $v \in \mathsf{U}$, *if* $\langle v, \mathsf{I} \rangle$ *is an extended Pollard feature structure then there exists exactly one* $\langle v', \mathsf{I}' \rangle \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$, *such that for each* $\delta \in \mathcal{D}^{\Sigma}$,

$v \in D_{\mathsf{I}}(\delta)$ *iff* $v' \in D_{\mathsf{I}'}(\delta)$.



## Strong Generative Capacity

We start with an auxiliary definition:

**Definition 3.** *For each signature* $\Sigma$, *for each* $\Sigma$ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, *for each* $v \in \mathsf{U}$,

$\langle v, \mathsf{I}_v \rangle$ *is the* **extended Pollard feature structure determined by** $v$ **in** $\mathsf{I}$ *iff*

$\mathsf{I}_v$ *is the* $\Sigma$ *interpretation generated by* $v$ *in* $\mathsf{I}$.

Pollard 1999 presents three equivalent definitions of the strong generative capacity of a grammar. All of them use the familiar relationships between the models of a grammar, the class of exhaustive models of a grammar, and the set of abstract feature structures that can be abstracted from them.

For the first definition of the strong generative capacity of a grammar, Pollard defines an abstraction function, $\mathrm{Abst}_{\mathsf{I}}$. For each entity $v$ in the interpretation $\mathsf{I}$, it contains pairs of $v$ and extended Pollard abstract feature structures whose root entities are indiscernible from $v$:



**Definition 4.** *For each signature* $\Sigma$, *for each* $\Sigma$ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$,

$\mathrm{Abst}_{\mathsf{I}}$ *is the binary relation on* $\mathsf{U} \times \mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$ *such that, for each* $v \in \mathsf{U}$, *for each* $\langle v', \mathsf{I}' \rangle \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$, $\langle v, \langle v', \mathsf{I}' \rangle \rangle \in \mathrm{Abst}_{\mathsf{I}}$ *iff*

*for the extended Pollard feature structure* $\langle v, \mathsf{I}_v \rangle$ *determined by* $v$ *in* $\mathsf{I}$, *for each* $\delta \in \mathcal{D}^{\Sigma}$, $v \in D_{\mathsf{I}_v}(\delta)$ *iff* $v' \in D_{\mathsf{I}'}(\delta)$.

Since each entity, $v$, in an interpretation, $\mathsf{I}$, determines exactly one extended Pollard feature structure, $\langle v, \mathsf{I}_v \rangle$, in the interpretation $\mathsf{I}$, it follows with PROPOSITION 1 that each $\mathrm{Abst}_{\mathsf{I}}$ is functional:

**Proposition 2.** *For each signature* $\Sigma$, *for each* $\Sigma$ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$, $\mathrm{Abst}_{\mathsf{I}}$ *is a total function from* $\mathsf{U}$ *to* $\mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$.

If $v$ is an entity in an interpretation $\mathsf{I}$, then I call $\mathrm{Abst}_{\mathsf{I}}(v)$ its *abstraction*. The strong generative capacity of a grammar is supposed to contain one isomorphic member to each utterance token in the natural language that is judged grammatical:

**Definition 5. [SGC 1]** *For each signature* $\Sigma$*,* SGC *is the total function from the set of grammars to* $Pow(\mathbb{AFS}^{\Sigma}_{\mathrm{eP}})$ *such that, for each* $\theta \subseteq \mathcal{D}^{\Sigma}$*,*

$$\mathsf{SGC}(\langle \Sigma, \theta \rangle) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}} \;\middle|\; \begin{array}{l} \textit{for some } \langle \Sigma, \theta \rangle \textit{ model} \\ \quad \mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle, \\ \textit{for some } v \in \mathsf{U}, \\ \alpha = \mathrm{Abst}_{\mathsf{I}}(v) \end{array} \right\}.$$

For each grammar $\Gamma$, Pollard calls the set $\mathsf{SGC}(\Gamma)$ the *strong generative capacity of* $\Gamma$.

Exhaustive models provide an obvious alternative to obtaining the SGC of a grammar by taking the abstract feature structures that can be abstracted from the entities in all models. It is clearly sufficient to consider only one model if that model is big enough in the sense of containing congruent copies of all possible configurations under entities in all models, and to take the set of abstract feature structures that can be abstracted from the entities in such a model.

Following Pollard 1999, we can exploit the existence of a canonical extended Pollard feature structure for each entity in an interpretation for a new way of characterizing exhaustive models:



**Definition 6.** *For each signature* $\Sigma$*, for each* $\theta \subseteq \mathcal{D}^{\Sigma}$*,*

$\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$ *is an* **exhaustive** $\langle \Sigma, \theta \rangle$ **model** *iff*

$\mathsf{I}$ *is a* $\langle \Sigma, \theta \rangle$ *model, and for each* $\langle \Sigma, \theta \rangle$ *model* $\mathsf{I}' = \langle \mathsf{U}', \mathsf{S}', \mathsf{A}', \mathsf{R}' \rangle$*, for each* $v' \in \mathsf{U}'$*, there exists a* $v \in \mathsf{U}$ *such that* $\mathrm{Abst}_{\mathsf{I}}(v) = \mathrm{Abst}_{\mathsf{I}'}(v')$*.*

The function that is defined next maps each $\Sigma$ interpretation I to the smallest set of extended Pollard abstract feature structures under $\Sigma$ that, for each extended Pollard feature structure determined by some $v$ in I, contains an element that is indiscernible from $\langle v, \mathsf{I}_v \rangle$:

**Definition 7.** *For each signature* $\Sigma$*,* ABST *is the large total function from the class of* $\Sigma$ *interpretations to* $Pow(\mathbb{AFS}^{\Sigma}_{\mathrm{eP}})$ *such that, for each* $\Sigma$ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$*,*

$$\mathsf{ABST}(\mathsf{I}) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}} \;\middle|\; \begin{array}{l} \textit{for some } v \in \mathsf{U}, \\ \alpha = \mathrm{Abst}_{\mathsf{I}}(v) \end{array} \right\}.$$

For each $\Sigma$ interpretation I, I call ABST(I) the *set of extended Pollard abstract feature structures determined*



*by* I. The function ABST leads to the second definition of the strong generative capacity of a grammar:

**Definition 8. [SGC 2]** *For each signature* $\Sigma$*,* SGC *is the total function from the set of grammars to* $Pow(\mathbb{AFS}^{\Sigma}_{\mathrm{eP}})$ *such that, for each* $\theta \subseteq \mathcal{D}^{\Sigma}$*,*

$$\mathsf{SGC}(\langle \Sigma, \theta \rangle) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}} \;\middle|\; \begin{array}{l} \textit{for each (equivalently some)} \\ \textit{exhaustive } \langle \Sigma, \theta \rangle \textit{ model } \mathsf{I}, \\ \alpha \in \mathsf{ABST}(\mathsf{I}) \end{array} \right\}.$$

Finally, the third way of defining the strong generative capacity of a grammar exploits the fact that an extended Pollard abstract feature structure, in contrast to traditional definitions of abstract feature structures, contains an interpretation of a signature. In particular, it contains the interpretation with canonical representations of the elements of the universe that is generated by its root entity. Following Pollard, we say that a grammar $\Gamma$ *generates* an extended Pollard abstract feature structure exactly if its interpretation is a $\Gamma$ model:



**Definition 9.** *For each signature* $\Sigma$*, for each* $\theta \subseteq \mathcal{D}^{\Sigma}$*, for each* $\Sigma$ *interpretation* $\mathsf{I} = \langle \mathsf{U}, \mathsf{S}, \mathsf{A}, \mathsf{R} \rangle$*, for each* $v \in \mathsf{U}$*,* $\langle \Sigma, \theta \rangle$ **generates** $\langle v, \mathsf{I} \rangle$ *iff*

$\langle v, \mathsf{I} \rangle \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}}$*, and*

I *is a* $\langle \Sigma, \theta \rangle$ *model.*

This leads to the last characterization of the SGC of a grammar as the set of all extended Pollard abstract feature structures whose interpretation is a model of a grammar $\Gamma$:

**Definition 10. [SGC 3]** *For each signature* $\Sigma$*,* SGC *is the total function from the set of grammars to* $Pow(\mathbb{AFS}^{\Sigma}_{\mathrm{eP}})$ *such that, for each* $\theta \subseteq \mathcal{D}^{\Sigma}$*,*

$$\mathsf{SGC}(\langle \Sigma, \theta \rangle) = \left\{ \alpha \in \mathbb{AFS}^{\Sigma}_{\mathrm{eP}} \;\middle|\; \langle \Sigma, \theta \rangle \textit{ generates } \alpha \right\}.$$

## Pollard and Sag 1994 in RSRL



## The Definition of an Admission Relation in RSRL

Suppose $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$ is a signature. I call each member of $\mathcal{A}^*$ a $\Sigma$ path, and write $\varepsilon$ for the empty path, the unique path of length zero.

**Definition 11.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$\mu$ *is a* **morph in** $\Sigma$ *iff*

$\mu$ *is a quadruple* $\langle \beta, \varrho, \lambda, \xi \rangle$,
$\beta \subseteq \mathcal{A}^*$,
$\varepsilon \in \beta$,
*for each* $\pi \in \mathcal{A}^*$, *for each* $\alpha \in \mathcal{A}$,
*if* $\pi\alpha \in \beta$ *then* $\pi \in \beta$,
$\varrho$ *is an equivalence relation over* $\beta$,
*for each* $\pi_1 \in \mathcal{A}^*$, *for each* $\pi_2 \in \mathcal{A}^*$, *for each* $\alpha \in \mathcal{A}$,
*if* $\pi_1\alpha \in \beta$ *and* $\langle \pi_1, \pi_2 \rangle \in \varrho$ *then* $\langle \pi_1\alpha, \pi_2\alpha \rangle \in \varrho$,
$\lambda$ *is a total function from* $\beta$ *to* $\mathcal{S}$,
*for each* $\pi_1 \in \mathcal{A}^*$, *for each* $\pi_2 \in \mathcal{A}^*$,
*if* $\langle \pi_1, \pi_2 \rangle \in \varrho$ *then* $\lambda(\pi_1) = \lambda(\pi_2)$,
*for each* $\pi \in \mathcal{A}^*$, *for each* $\alpha \in \mathcal{A}$,



*if* $\pi\alpha \in \beta$ *then* $\mathcal{F}\langle \lambda(\pi), \alpha \rangle$ *is defined and* $\lambda(\pi\alpha) \sqsubseteq \mathcal{F}\langle \lambda(\pi), \alpha \rangle$,
*for each* $\pi \in \mathcal{A}^*$, *for each* $\alpha \in \mathcal{A}$,
*if* $\pi \in \beta$ *and* $\mathcal{F}\langle \lambda(\pi), \alpha \rangle$ *is defined then* $\pi\alpha \in \beta$,
$\xi \subseteq \mathcal{R} \times \overline{\beta}^*$,
*for each* $\rho \in \mathcal{R}$, *for each* $\pi_1 \in \overline{\beta}$,..., *for each* $\pi_n \in \overline{\beta}$,
*if* $\langle \rho, \pi_1, \ldots, \pi_n \rangle \in \xi$ *then* $n = \mathcal{AR}(\rho)$, *and*
*for each* $\rho \in \mathcal{R}$, *for each* $\pi_1 \in \overline{\beta}$, ..., *for each* $\pi_n \in \overline{\beta}$, *for each* $\pi'_1 \in \overline{\beta}$, ..., *for each* $\pi'_n \in \overline{\beta}$,
*if* $\langle \rho, \pi_1, \ldots, \pi_n \rangle \in \xi$, *and*
*for each* $i \in \{1, \ldots, n\}$,
$\pi_i \in \beta$ *and* $\langle \pi_i, \pi'_i \rangle \in \varrho$, *or*
*for some* $m \in \mathbb{N}$,
$\pi_i \in \beta^*$,
$\pi_i = \langle \pi_{i_1}, \ldots, \pi_{i_m} \rangle$,
$\pi'_i = \langle \pi'_{i_1}, \ldots, \pi'_{i_m} \rangle$, *and*
$\langle \pi_{i_1}, \pi'_{i_1} \rangle \in \varrho, \ldots, \langle \pi_{i_m}, \pi'_{i_m} \rangle \in \varrho$,
*then* $\langle \rho, \pi'_1, \ldots, \pi'_n \rangle \in \xi$.

Suppose $\Sigma$ is a signature and $\mu = \langle \beta, \varrho, \lambda, \xi \rangle$ is a $\Sigma$ morph. I call $\beta$ the *basis set in* $\mu$, $\varrho$ the *re-entrancy relation in* $\mu$, $\lambda$ the *label function in* $\mu$, and $\xi$ the *relation extension in* $\mu$. I write $\mathcal{M}_\Sigma$ for the set of $\Sigma$ morphs.



The $\Sigma$ morphs are a straightforward extension of the abstract feature structures of Moshier 1988.

**Definition 12.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* $\pi \in \mathcal{A}^*$, *for each* $\langle \pi_1, \ldots, \pi_n \rangle \in \mathcal{A}^{**}$,

$\pi\langle \pi_1, \ldots, \pi_n \rangle$ *is an abbreviatory notation for* $\langle \pi\pi_1, \ldots, \pi\pi_n \rangle$.

**Definition 13.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* $\mu = \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma$, *for each* $\pi \in \mathcal{A}^*$,

$\beta/\pi = \{\pi' \in \mathcal{A}^* \mid \pi\pi' \in \beta\}$,

$\varrho/\pi = \{\langle \pi_1, \pi_2 \rangle \in \mathcal{A}^* \times \mathcal{A}^* \mid \langle \pi\pi_1, \pi\pi_2 \rangle \in \varrho\}$,

$\lambda/\pi = \{\langle \pi', \sigma \rangle \in \mathcal{A}^* \times \mathcal{S} \mid \langle \pi\pi', \sigma \rangle \in \lambda\}$,

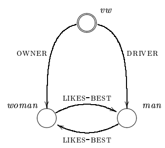
$\xi/\pi =$
$\{\langle \rho, \pi_1, \ldots, \pi_n \rangle \in \mathcal{R} \times \overline{(\beta/\pi)}^* \mid \langle \rho, \pi\pi_1, \ldots, \pi\pi_n \rangle \in \xi\}$, *and*

$\mu/\pi = \langle \beta/\pi, \varrho/\pi, \lambda/\pi, \xi/\pi \rangle$.

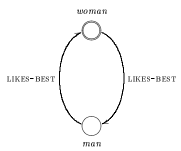
If $\Sigma$ is a signature, $\mu$ is a $\Sigma$ morph and $\pi$ is a $\Sigma$ path then I call $\mu/\pi$ the $\pi$ *reduct of* $\mu$.

For example, the OWNER reduct of the $\Sigma$ morph in (1a) is the $\Sigma$ morph in (1b):

(1)a.




b.




If a $\Sigma$ path $\pi$ is in the basis set of the $\Sigma$ morph $\mu$ then the $\pi$ reduct of $\mu$ is also a $\Sigma$ morph:

**Proposition 3.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* $\mu = \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma$, *for each* $\pi \in \mathcal{A}^*$,

*if* $\pi \in \beta$ *then* $\mu/\pi \in \mathcal{M}_\Sigma$.



DEFINITION 14 provides a necessary substitute for the variable assignments in interpretations for morph satisfaction. Instead of entities or chains of entities, variables are assigned paths or tuples of paths:

**Definition 14.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$\mathrm{I}_\Sigma$ *is the set of total functions from* $\mathcal{VAR}$ *to* $\overline{\mathcal{A}^*}$.

Let $\Sigma$ be a signature. Then I call each member of $\mathrm{I}_\Sigma$ a $\Sigma$ *insertion*. Given a $\Sigma$ insertion that assigns the $\Sigma$ path $\pi$ to the variable $v$, a $\Sigma$ morph $\mu = \langle \beta, \varrho, \lambda, \xi \rangle$ satisfies the $\Sigma$ formula '$v \sim \sigma$' only if $\langle \pi, \sigma \rangle \in \lambda$.

For the definition of morph satisfaction, we finally need the three auxiliary definitions that follow next:

**Definition 15.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$,

$\mathrm{T}_\Sigma$ *is the smallest partial function from* $\overline{\mathcal{A}^*} \times \widehat{\mathcal{A}}$ *to* $\overline{\mathcal{A}^*}$ *such that,*

*for each* $\pi \in \mathcal{A}^*$, *for each* $\alpha \in \mathcal{A}$,
$\mathrm{T}_\Sigma(\pi, \alpha) = \pi\alpha$,



*for each* $\langle \pi_0, \ldots, \pi_n \rangle \in \mathcal{A}^{**}$,
$\mathrm{T}_\Sigma(\langle \pi_0, \ldots, \pi_n \rangle, \dagger) = \pi_0$,
$\mathrm{T}_\Sigma(\langle \pi_0, \ldots, \pi_n \rangle, \triangleright) = \langle \pi_1, \ldots, \pi_n \rangle$.

**Definition 16.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* $\iota \in \mathrm{I}_\Sigma$,

$\Pi^\iota_\Sigma$ *is the smallest partial function from* $\mathcal{T}^\Sigma$ *to* $\overline{\mathcal{A}^*}$ *such that*

$\Pi^\iota_\Sigma(:) = \varepsilon$,
*for each* $v \in \mathcal{VAR}$, $\Pi^\iota_\Sigma(v) = \iota(v)$, *and*
*for each* $\tau \in \mathcal{T}^\Sigma$, *for each* $\alpha \in \widehat{\mathcal{A}}$, $\Pi^\iota_\Sigma(\tau\alpha) = \mathrm{T}_\Sigma(\Pi^\iota_\Sigma(\tau), \alpha)$.

Suppose $\Sigma$ is a signature, and $\iota$ is a $\Sigma$ insertion. Then I call each $\Pi^\iota_\Sigma$ the *path insertion function under* $\iota$ *in* $\Sigma$.

**Definition 17.** *For each signature* $\Sigma$, *for each* $\langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma$,

$\hat{\varrho}$ *is the smallest subset of* $\overline{\beta} \times \overline{\beta}$ *such that*

$\varrho \subseteq \hat{\varrho}$, *and*
*for each* $\pi_1 \in \beta, \ldots, \pi_n \in \beta$, *for each* $\pi'_1 \in \beta, \ldots, \pi'_n \in \beta$,



*if* $\langle \pi_1, \pi'_1 \rangle \in \varrho$, ..., *and* $\langle \pi_n, \pi'_n \rangle \in \varrho$
*then* $\langle \langle \pi_1, \ldots, \pi_n \rangle, \langle \pi'_1, \ldots, \pi'_n \rangle \rangle \in \hat{\varrho}$,

$\hat{\lambda}$ *is the total function from* $\overline{\beta}$ *to* $\widehat{\mathcal{S}}$ *such that*

*for each* $\pi \in \beta$, $\hat{\lambda}(\pi) = \lambda(\pi)$,
*for each* $\pi_1 \in \beta$, ..., *for each* $\pi_n \in \beta$,

$$\hat{\lambda}(\langle \pi_1, \ldots, \pi_n \rangle) = \begin{cases} echain & \text{if } n = 0, \\ nechain & \text{if } n > 0. \end{cases}$$

For each signature $\Sigma$ and for each $\Sigma$ insertion $\iota \in \mathrm{I}_\Sigma$, the morph satisfaction function under $\iota$ in $\Sigma$, $\Delta^\iota_\Sigma$ is defined inductively for all $\Sigma$ formulae.

**Definition 18.** *For each signature* $\Sigma = \langle \mathcal{G}, \sqsubseteq, \mathcal{S}, \mathcal{A}, \mathcal{F}, \mathcal{R}, \mathcal{AR} \rangle$, *for each* $\iota \in \mathrm{I}_\Sigma$, $\Delta^\iota_\Sigma$ *is the total function from* $\mathcal{D}^\Sigma$ *to* $Pow(\mathcal{M}_\Sigma)$ *such that*

*for each* $\tau \in \mathcal{T}^\Sigma$, *for each* $\sigma \in \widehat{\mathcal{G}}$,

$$\Delta^\iota_\Sigma(\tau \sim \sigma) = \left\{ \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma \,\middle|\, \begin{array}{l} \Pi^\iota_\Sigma(\tau) \text{ is defined, and,} \\ \text{for some } \sigma' \in \widehat{\mathcal{S}}, \\ \quad \langle \Pi^\iota_\Sigma(\tau), \sigma' \rangle \in \hat{\lambda} \text{ and} \\ \quad \sigma' \mathrel{\widehat{\sqsubseteq}} \sigma \end{array} \right\},$$

*for each* $\tau_1 \in \mathcal{T}^\Sigma$, *for each* $\tau_2 \in \mathcal{T}^\Sigma$,

$$\Delta^\iota_\Sigma(\tau_1 \approx \tau_2) = \left\{ \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma \,\middle|\, \begin{array}{l} \Pi^\iota_\Sigma(\tau_1) \textit{ is defined,} \\ \Pi^\iota_\Sigma(\tau_2) \textit{ is defined, and} \\ \langle \Pi^\iota_\Sigma(\tau_1), \Pi^\iota_\Sigma(\tau_2) \rangle \in \hat{\varrho} \end{array} \right\},$$

*for each* $\rho \in \mathcal{R}$, *for each* $v_1 \in \mathcal{VAR}$, ..., *for each* $v_n \in \mathcal{VAR}$,

$$\Delta^\iota_\Sigma(\rho(v_1, \ldots, v_n)) = \left\{ \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma \,\middle|\, \langle \rho, \iota(v_1), \ldots, \iota(v_n) \rangle \in \xi \right\},$$

*for each* $v \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$,

$$\Delta^\iota_\Sigma(\exists v\, \delta) = \left\{ \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma \,\middle|\, \begin{array}{l} \textit{for some } \pi \in \overline{\beta}, \\ \quad \langle \beta, \varrho, \lambda, \xi \rangle \in \Delta^{\iota[\frac{\pi}{v}]}_\Sigma(\delta) \end{array} \right\},$$

*for each* $v \in \mathcal{VAR}$, *for each* $\delta \in \mathcal{D}^\Sigma$,

$$\Delta^\iota_\Sigma(\forall v\, \delta) = \left\{ \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma \,\middle|\, \begin{array}{l} \textit{for each } \pi \in \overline{\beta}, \\ \quad \langle \beta, \varrho, \lambda, \xi \rangle \in \Delta^{\iota[\frac{\pi}{v}]}_\Sigma(\delta) \end{array} \right\},$$

*for each* $\delta \in \mathcal{D}^\Sigma$,

$$\Delta^\iota_\Sigma(\neg\delta) = \mathcal{M}_\Sigma \backslash \Delta^\iota_\Sigma(\delta),$$



*for each* $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$\Delta^\iota_\Sigma([\delta_1 \wedge \delta_2]) = \Delta^\iota_\Sigma(\delta_1) \cap \Delta^\iota_\Sigma(\delta_2),$$

*for each* $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$\Delta^\iota_\Sigma([\delta_1 \vee \delta_2]) = \Delta^\iota_\Sigma(\delta_1) \cup \Delta^\iota_\Sigma(\delta_2), \textit{ and}$$

*for each* $\delta_1 \in \mathcal{D}^\Sigma$, *for each* $\delta_2 \in \mathcal{D}^\Sigma$,

$$\Delta^\iota_\Sigma([\delta_1 \rightarrow \delta_2]) = (\mathcal{M}_\Sigma \backslash \Delta^\iota_\Sigma(\delta_1)) \cup \Delta^\iota_\Sigma(\delta_2).$$

Let $\Sigma$ be a signature, $\mu$ a $\Sigma$ morph, $\iota$ a $\Sigma$ insertion and $\delta$ a $\Sigma$ formula. I call $\Delta^\iota_\Sigma$ the *morph satisfaction function under* $\iota$ *in* $\Sigma$, and say $\mu$ *satisfies* $\delta$ *under* $\iota$ *in* $\Sigma$ if and only if $\mu \in \Delta^\iota_\Sigma(\delta)$.

**Definition 19.** *For each signature* $\Sigma$,

$M_\Sigma$ *is the total function from* $Pow(\mathcal{D}^\Sigma_0)$ *to* $Pow(\mathcal{M}_\Sigma)$ *such that for each* $\theta \subseteq \mathcal{D}^\Sigma_0$,

$$M_\Sigma(\theta) = \left\{ \langle \beta, \varrho, \lambda, \xi \rangle \in \mathcal{M}_\Sigma \,\middle|\, \begin{array}{l} \textit{for each } \pi \in \beta, \\ \textit{for each } \iota \in \mathrm{I}_\Sigma, \textit{ and} \\ \textit{for each } \delta \in \theta, \\ \quad \langle \beta, \varrho, \lambda, \xi \rangle / \pi \in \Delta^\iota_\Sigma(\delta) \end{array} \right\}.$$



Let $\Sigma$ be a signature. I call $M_\Sigma$ the *morph admission function in* $\Sigma$. A $\Sigma$ morph $\mu$ is in the set of $\Sigma$ morphs admitted by a $\Sigma$ theory $\theta$ exactly if $\mu$ and every possible $\pi$ reduct of $\mu$ satisfy every description in $\theta$.



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