A Proof Theoretic Investigation of Prediction in HPSG

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Chapter 1

Introduction

[PS94] founds its Head-Driven Phrase Structure Grammar (HPSG) on specific methodological assumptions which, [Kin] argues, demand a formalism that meets certain requirements. I additionally motivate the desirability of a sound and complete calculus for such a formalism and show that, although the HPSG formalism of [Kin] meets all of [Kin]'s requirements, a sound and complete calculus for that formalism is logically inconceivable.

In this chapter, I motivate the usefulness of a sound and complete calculus for any HPSG formalism and claim that such a calculus cannot be provided for the HPSG formalism of [Kin]. The chapter comprises three sections. In the first section, I outline, much in accordance with [Kin], certain properties of the deductive method in science and conclude that any science proceeding in accordance with the deductive method demands a formalism that must meet certain requirements and could advantageously meet certain desiderata — a sound and complete calculus being one of them. In the second section, I demonstrate that HPSG is intended to be a deductive science and infer that a sound and complete calculus for the HPSG formalism of [Kin] is consequently desirable. I nevertheless claim that this goal is unachievable. In the third section, I preview the method I use to justify my claim.

1.1 Deductive Science

Empirical science is the description of observable regularities among given tangible phenomena. Rather than simply listing the regularities, the practitioners of the deductive method proceed by examining the properties of the tangible phenomena and formulating general hypotheses and theories about the nature of these properties, which hypothetically gives rise to the regularities observed. If no properties advantageous to 'explain' the regularities can be identified by observation and experiment, they are postulated. In a direct sense, the theories talk about the properties, but in an indirect sense, the theories are understood to describe the empirical domain by means of logical deduction: Generally, if a theory is true then a conclusion logically deduced from the theory is also true, and particularly, if the nature of the properties of the tangible phenomena is exactly as the theory asserts it to be and this nature gives rise to observable regularities among the phenomena, then the regularities are exactly as the conclusions logically deduced from the theory assert them to be. A (possibly infinite) number of particular regularities among tangible phenomena may thus be described by a finite number of general statements about the properties of the phenomena, even regularities that have hitherto not been observed. For example, the postulation that matter has an internal structure consisting of (unobservable at the time of the postulation) protons and electrons and the hypothesis that matters with different numbers of protons and electrons react differently not only described in a general fashion the already observed regularities that some matters reacted differently than others, but also predicted reactions of matters which had not been observed yet.

However, the postulation of properties whose existence can not be experimentally verified or falsified may seem unsatisfactory or even unscientific to a practitioner of a science; an alternative path is not to claim that the phenomena themselves possess intangible properties, but rather to introduce intermediating modelling structures which both stand in a one-to-one correspondence with the phenomena by means of sharing certain observable aspects and additionally possess the postulated properties. Clearly, since the modelling structures are an artificial construct to begin with, they can also be stipulated to possess any properties that might prove to be utile to predict the regularities among the phenomena that they model, without the need to justify the actual *reality* of such properties. The hypotheses and theories then refer to the modelling structures, not to the observable phenomena themselves. Via the assumed one-to-one correspondence between the two, however, any statement about the modelling structures by the theory is then taken to correspond to a statement about the observable phenomena.

Note that the difference between the claim that tangible phenomena possess certain properties and the use of modelling structures with such properties is merely a philosophical nicety under the assumption that the domain of a deductive science is understood to be only *observable* regularities among the tangible components of the phenomena, not the intangible properties that 'explain' the regularities. In this sense, both approaches are equivalent. Taking a linguistic example, to merely characterize the set of syntactically well-formed natural language expressions, it is irrelevant whether (intangible) phrase structure is assumed to be part of natural language expressions or to be part of modelling structures that stand in a one-to-one correspondence with natural language expressions; the observational — though maybe not the explanatory — value of a theory about phrase structure clearly remains the same in either case.

To arrive at a description of an empirical domain under the deductive method calls then for the arrival at a theory whose logical conclusions veraciously describe the domain. To this end, it has to be *tested* whether the conclusions of the theory veraciously describe the domain in order to enable the evaluation of the theory, the comparison with its competitors and the choice from a candidate set of a theory that best describes the domain for the time being. [Pop59] suggests that

> the method of critically testing theories, and selecting them according to the results of tests, always proceeds on the following lines. From a new idea, put up tentatively, and not yet justified in any way — an anticipation, a hypothesis, a theoretical system, or what you will — conclusions are drawn by means of logical deduction. These conclusions are then compared with one another and with other relevant statements, so as to find what logical relations (such as equivalence, derivability, compatibility, or incompatibility) exist between them.
>
> We may if we like distinguish four different lines along which the testing of a theory could be carried out. First there is the logical comparison of

> the conclusions among themselves, by which the internal consistency of the system is tested. Secondly, there is the investigation of the logical form of the theory, with the object of determining whether it has the character of an empirical or scientific theory, or whether it is, for example, tautological. Thirdly, there is the comparison with other theories, chiefly with the aim of determining whether the theory would constitute a scientific advance should it survive our various tests. And finally, there is the testing of the theory by way of empirical applications of the conclusions which can be derived from it.
>
> The purpose of this last test is to find out how far the new consequences of the theory — whatever may be new in what it asserts — stand up to the demands of practice, whether raised by purely scientific experiments, or by practical technological applications. Here too the procedure of testing turns out to be deductive. With the help of other statements, previously accepted, certain singular statements — which we may call 'predictions' — are deduced from the theory; especially predictions that are easily testable or applicable. From among these statements, those are selected which are not derivable from the current theory, and more especially those which the current theory contradicts. Next we seek a decision as regards these (and other) derived statements by comparing them with the results of practical applications and experiments. If this decision is positive, that is, if the singular conclusions turn out to be acceptable, or *verified*, then the theory has, for the time being, passed its test: we have found no reason to discard it. But if the decision is negative, or in other words, if the conclusions have been *falsified*, then their falsification also falsifies the theory from which they were logically deduced.
>
> It should be noticed that a positive decision can only temporarily support the theory, for subsequent negative decisions may always overthrow it. So long as a theory withstands detailed and severe tests and is not superseded by another theory in the course of scientific progress, we may say that it has 'proved its mettle' or that it is *'corroborated'*.
>
> [Pop59, pages 32 and 33]

Although a number of deductive sciences certainly progress without strictly adhering to the above mentioned and somewhat idealized method of evaluating their theories, it nevertheless remains clear that any science describing an empirical domain in accordance with the deductive method requires a *formalism*, i.e. at least (i) a means to enable the expression of theories and (ii) a definition of a deduction relation, or *prediction relation*, in which theories and experimentally verifiable or falsifiable assertions about observable regularities among tangible phenomena stand. A theory may then be tested by testing those of its conclusions that make assertions about observables; if the assertions made by the conclusions of the theory are found to be true then the theory is satisfactory to the extent that it has been tested, but if the assertions made by the conclusions are found to be false then the theory is simply false.

Yet, a formalism that only satisfies requirements (i) and (ii) is clearly less than optimal: Establishing that a theory predicts an assertion demands a *demonstration* that is clear and, in particular, *convincing* enough to be accepted by the scientific community, but a formalism that only gives a definition of the prediction relation provides no guidance of how to obtain such a convincing demonstration, or proof.

Clearly, some argument in a natural language may in principle be a generally convincing demonstration, but this is by no means guaranteed for just any such argument. Thus, in addition to the above requirements it is desirable — though maybe not obligatory — that a formalism possess further properties that facilitate the location of such convincing demonstrations. I consider two such properties in turn.

Better would be a formalism that provides not only a definition of a prediction relation, but also a *sound and complete calculus* for the relation, i.e. a language that provides (i) some expression interpreted as a demonstration that a theory predicts an assertion if and only if the theory predicts the assertion and (ii) a method to scrutinize in finite time for any candidate demonstration whether it is or is not an expression of the language. On account of (i), two things follow immediately: Firstly, each expression of a sound and complete calculus for a prediction relation can clearly be construed as a convincing demonstration that some theory predicts some assertion, and secondly, the calculus contains all demonstrations needed for showing that a given theory predicts a given assertion. Moreover, on account of (ii), anyone who accepts the calculus as convincing may decide if or if not a given candidate demonstration is convincing by scrutinizing if or if not it is an expression of the calculus. While a sound and complete calculus for a prediction relation does not liberate a scientist from supplying a demonstration that a given theory predicts a given assertion, it does liberate him from establishing that a demonstration he supplies convinces.

Better yet would be a formalism that provides not only a definition of a prediction relation and a sound and complete calculus for the relation, but also a *decidability method* for the relation, i.e. a general method that, for each (finite) theory and each assertion, mechanistically determines in finite time if or if not the calculus furnishes a convincing demonstration that the theory predicts the assertion. For each theory and assertion, a positive (negative) answer of such a method can clearly be construed as a convincing demonstration that the theory predicts (does not predict) the assertion. Thus, a decidability method for a prediction relation does not only liberate a scientist from establishing that a given demonstration that a theory predicts an assertion is convincing, but also liberates him from finding a convincing demonstration, since the demonstration is determined algorithmically instead.

In sum, any deductive science *must* provide a formalism, a means to enable the expression of theories and a definition of a prediction relation such that theories predict assertions with experimentally verifiable or falsifiable content (possibly by employing modelling structures). However, such a formalism barely meets the needs of a deductive science and *could* advantageously provide a sound and complete calculus for the prediction relation, or, even better, a decidability method for the prediction relation to facilitate the evaluation of its theories.

1.2 Head-Driven Phrase Structure Grammar

Although [PS94] lacks an explicit statement, the HPSG literature makes it nevertheless plausible to assume that HPSG is intended to proceed in accordance with the deductive method.

Firstly, [PS87] concisely summarizes the general subgoal of the research program to which HPSG belongs as

> to develop a class of mathematical structures suitable for modelling natural

> language phenomena, together with a logical formalism interpreted by these structures, in terms of which it is possible to formulate precise linguistic theories with determinate empirical consequences. [PS87, page v]

Somewhat more in detail, [PS94] explicates and illustrates how formalized theories about modelling structures may describe an empirical domain.

> In any mathematical theory about an empirical domain, the phenomena of interest are *modelled* by mathematical structures, certain aspects of which are conventionally understood as corresponding to observables of the domain. The theory itself does not talk directly about the empirical phenomena; instead, it talks about, or is *interpreted by*, the modelling structures. Thus the predictive power of the theory arises from the conventional correspondence between the model and the empirical domain.
>
> An informal theory is one that talks about the model in natural language, say a technical dialect of English, German, or Japanese. But as theories become more complicated and their empirical consequences less straightforwardly apparent, the need for formalization arises. In cases of extreme formalization, of course, the empirical hypotheses are cast as a set of axioms in a logical language, where the modelling structures serve as the intended interpretations of expressions in the logic.
>
> For example, in one kind of standard model of celestial mechanics, the positions and velocities of bodies subject to mutual gravitation are represented by vectors in a higher-dimensional Euclidean space ('phase space'), the masses of the bodies by positive real numbers, and their motions by paths along certain smooth vector fields ('flows') on the space. Of course such a model is not the same thing as what it models (e.g. the solar system), but certain formal properties of such a model may represent aspects of the solar system of interest to a physicist. In a formal theory based on such a model, the underlying logic is just a standard first-order language (e.g. the language of Zermelo-Fraenkel set theory), and the axioms are certain systems of differential equations (e.g. Hamiltonian systems) that the flows are required to satisfy. An observed motion of the solar system is then predicted by the theory insofar as it agrees—under the conventional correspondence—with an admissible flow (i.e. one that satisfies the equations). This state of affairs is summarized in (1):

SOLAR SYSTEM PICTURE HERE

[PS94, pages 6 and 7]

[PS94] intends to proceed along these lines in HPSG to describe the empirical domain of possible linguistic objects. The theories employed must have "determinate empirical consequences".

> In our view, a linguistic theory should bear exactly the same relation to the empirical domain of natural language, namely, the universe of possible linguistic objects, as a mathematical theory of celestial mechanics bears to the possible motions of n-body systems. Thus we insist on being explicit as to what sorts of constructs are assumed (i.e. what ontological categories of linguistic objects we suppose to populate the empirical domain) and on being mathematically rigorous as to what structures are used to model them. Moreover, we require that the theory itself actually count as a theory in the technical sense of precisely characterizing those modelling structures that are regarded as admissible or well-formed (i.e. corresponding to those imaginable linguistic objects that are actually predicted to be possible ones). This does not mean that the empirical hypotheses must be rendered in a formal logic as long as their content can be made clear and unambiguous in natural language (the same holds true in mathematical physics), but in principle they must be capable of being so rendered. Unless these criteria are satisfied, an enterprise purporting to be a theory cannot have any determinate empirical consequences. Rather, our position is the same as the one advocated by Chomsky (1957: 5):
>
>> Precisely constructed models for linguistic structure can play an important role, both negative and positive, in the process of discovery itself. By pushing a precise but inadequate formulation to an unacceptable conclusion, we can often expose the exact source of this inadequacy and, consequently, gain a deeper understanding of the linguistic data. More positively, a formalized theory may automatically provide solutions for many problems other than those for which it was explicitly designed. Obscure and intuition-bound notions can neither lead to absurd conclusion nor provide new and correct ones, and hence they fail to be useful in two important respects. I think that some of those linguists who have questioned the value of precise and technical development of linguistic theory may have failed to recognize the productive

> potential in the method of rigorously stating a proposed theory and applying it strictly to linguistic material with no attempt to avoid unacceptable conclusions by *ad hoc* adjustments or loose formulation.
>
> [PS94, pages 7 and 8]

[PS94] uses sorted feature structures as modelling structures for (types of) natural language expressions. The expressions that correspond to modelling structures admitted by the theory are the possible linguistic objects described by the theory.

> In HPSG, the modelling domain—the analog of the physicist's flows—is a system of *sorted feature structures* (Moshier 1988; Pollard and Moshier 1990), that are intended to stand in a one-to-one relation with types of natural language expressions and their subparts. The role of the linguistic theory is to give a precise specification of which feature structures are to be considered admissible; the types of linguistic entities that correspond to the admissible feature structures constitute the predictions of the theory.
> [PS94, page 8]

> Our research program thus assumes the following three-way relation connecting theory, model, and the empirical domain of language:
>
> LANGUAGE PICTURE HERE
>
> [PS94, pages 8 and 9]

In sum, an HPSG theory describes a natural language expression if and only if the sorted feature structure that stands in a one-to-one correspondence with the expression is admitted by the theory. Moreover, the theory must have determinate empirical consequences.

However, note that if an HPSG theory describes an object with components intangible to the testing methods available then the theory cannot have determinate empirical consequences in the strong sense that the existence of *the object* can be verified or falsified by means of testing. Rather, the testing method at best provides a means to decide on the existence of some object which agrees on the testable components with the object described by the theory, in other words, the method at best provides a means to decide on the existence of the *testable aspects* of the described object. By contrast, if the fact that an HPSG theory describes an object is not interpreted as a statement that the object exists, but reinterpreted in the sense that the testable aspects of the object exist, then the testability of the statement and thus the theory is ensured.

By way of example, [PS94] seems to accept grammaticality judgments of language users as a method to test candidate natural language expressions by asserting that they are "generally taken as the primary data to be accounted for by the theory" [PS94, page 11]. Under the assumption that grammaticality judgments may decide on the existence of linguistic objects with a certain phonological component, but not a certain phrase structure or Cooper storage component, it cannot be determined if or if not a given natural language object has a certain phrase structure or Cooper storage component. Nevertheless, what *can* presumably be determined is if or if not the object has a well-formed phonological component. Thus, interpreting the fact that an HPSG theory describes an object with a certain phonological, phrase structure and Cooper storage component as a statement that *the object* exists may prevent the testability of the theory, but interpreting the same fact as a statement that *the (testable) phonological aspect* exists enables the testability of the theory. But a phonological aspect exists in a natural language if and only if *some object* with such component exists. In essence, then, a demonstration that a given natural language expression is (in the reinterpreted sense) described by a given HPSG theory demands a demonstration that *some* feature structure that agrees on all testable aspects with the expression is admitted by the theory.

All in all, HPSG describes the empirical domain of natural language by (i) the expression of theories and (ii) the prediction of experimentally verifiable or falsifiable assertions about the existence of linguistic objects. Consequently, HPSG is intended to be a deductive science. As such, HPSG *requires* a formalism to make the deductive method possible and *could utilize* a sound and complete calculus, or, even better, a decidability method for the prediction relation.

[Kin] argues that the formal logic of [Kin89] — speciate re-entrant logic — extended with a prediction relation[1] provides a formalism for HPSG. However, a straightforward corollary of [KSA] states that the prediction relation of this formalism does not meet the strong desideratum of decidability. I moreover claim that even the weaker desideratum of a sound and complete calculus for this prediction relation is logically inconceivable.

1.3 Preview

The remaining chapters are devoted to the justification of my claim at the end of the previous section.

In chapter 2, I lay the mathematical foundations for my argument by outlining some basic notions from computation theory, a standard version of first-order predicate logic (FOL) and the [Kin] formalism for HPSG — speciate re-entrant logic (SRL) extended with a prediction relation.

In chapter 3, I assign each limited SRL signature, nonempty SRL interpretation and SRL description an FOL analog. This allows me to establish a correlation between SRL non-prediction[2] and FOL inference.

[1] Prediction is not part of the formulation of speciate re-entrant logic in [Kin89].

[2] A theory non-predicts an assertion iff the theory does not predict the assertion.

In chapter 4, I first argue that if a sound and complete calculus for a relation exists then the relation is Turing acceptable (i.e. recursively enumerable). I then use the correlation from chapter 3 to transfer the Turing acceptability of FOL inference to SRL non-prediction; and a result from [KSA] to prove that SRL prediction is not Turing decidable (i.e. not recursive). From this I conclude that SRL prediction is not Turing acceptable, hence that there is no sound and complete calculus for SRL prediction.

Chapter 2

Prerequisites

An enterprise purporting to formally demonstrate a nontrivial result should precisely state at least the formal notions involved in the demonstration, even if these notions are *in principle* common knowledge. I therefore take this chapter to introduce the formal prerequisites for chapters 3 and 4. The chapter comprises three sections. In the first section, I recapitulate some basic notions from computation theory. In the second section, I review the standard version of first-order predicate logic in [Bar77], but with a number of small notational changes more in conformity with the presentation of the third section. In the third section, I outline the [Kin] formalism for HPSG, in major parts cited from the presentation of [Kin]. In either case, I make little effort to motivate the intuitions underlying the formal notions, but rather refer the reader to any standard textbook for an introduction to computation theory[1] and FOL[2], and to [Kin] for a more thorough motivation that SRL extended with a prediction relation provides a formalism for HPSG as envisaged in [PS94].

2.1 Computation Theory

I presuppose familiarity with the notion of a *Turing machine over an alphabet* and merely recapitulate some related notions (basically cited from [LP81]) relevant for later chapters.

An alphabet is a finite set of symbols.

Definition 1 *For each set $\mathcal{A}$:*
$\mathcal{A}$ is an **alphabet** *iff $\mathcal{A}$ is finite.*

If $\mathcal{A}$ is an alphabet then I call each member of $\mathcal{A}$ a **symbol** from $\mathcal{A}$, each finite (including the empty) sequence of symbols from $\mathcal{A}$ a **string** from $\mathcal{A}$ and write $\mathcal{A}^*$ for the set of all strings from $\mathcal{A}$. A language over an alphabet is a set of strings from the alphabet.

Definition 2 *For each alphabet $\mathcal{A}$, for each L: L is a* **language** *over $\mathcal{A}$ iff $L \subseteq \mathcal{A}^*$.*

There exist well-known notions of a Turing machine accepting, enumerating and deciding a string or language.

[1] For example [LP81] or [HU79].
[2] For example [Bar77].

Definition 3 *For each alphabet $\mathcal{A}$, for each Turing machine $\mathfrak{M}$:*

For each string w from $\mathcal{A}$: $\mathfrak{M}$ **accepts** *w iff $\mathfrak{M}$ halts on input w.*
For each language L over $\mathcal{A}$: $\mathfrak{M}$ **accepts** *L iff $L = \{w \in \mathcal{A}^* | \mathfrak{M} \text{ accepts } w\}$.*
For each language L over $\mathcal{A}$:

$\mathfrak{M}$ **enumerates** *L iff $\mathfrak{M}$ outputs exactly the strings in L (one after another).*

For each language L over $\mathcal{A}$:

$\mathfrak{M}$ **decides** L

iff for each string w from $\mathcal{A}$:

if $w \in L$
then $\mathfrak{M}$ outputs $\top$ on input w
else $\mathfrak{M}$ outputs $\perp$ on input w.

Also, there exist well-known notions of a language being Turing acceptable, Turing enumerable and Turing decidable.

Definition 4 *For each alphabet $\mathcal{A}$, for each language L over $\mathcal{A}$, for each Turing machine $\mathfrak{M}$:*

L is **Turing acceptable**, **Turing enumerable** *or* **Turing decidable**
iff for some Turing machine $\mathfrak{M}$:

$\mathfrak{M}$ accepts, enumerates or decides L, respectively.

[LP81, pages 278 and 279] shows that a language is Turing decidable iff both it and its complement are Turing acceptable, and [LP81, page 280] shows that a language is Turing acceptable iff the language is Turing enumerable.

Proposition 1 *For each alphabet $\mathcal{A}$, for each language L over $\mathcal{A}$:*
L is Turing decidable iff L is Turing acceptable and $\mathcal{A}^* \backslash L$ is Turing acceptable.

Proposition 2 *For each alphabet $\mathcal{A}$, for each language L over $\mathcal{A}$:*
L is Turing acceptable iff L is Turing enumerable.

2.2 First-Order Predicate Logic

A signature comprises three sets of symbols and a function that encodes certain conditions on the interpretation of the symbols.

Definition 5 *Quadruple $\langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$ is an* **FOL signature**
iff $\mathcal{C}$ is a set,
$\mathcal{F}$ is a set,
$\mathcal{R}$ is a set,
Ar is a total function from $\mathcal{F} \cup \mathcal{R}$ to $\mathbb{N}^+$.

If $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$ is an FOL signature then I call each member of $\mathcal{C}$ a **constant** from Σ, each member of $\mathcal{F}$ a **function symbol** from Σ, each member of $\mathcal{R}$ a **relation symbol** from Σ and Ar the **arity function** of Σ. For notational facility, I assume that each element of $\{x_i | i \in \mathbb{N}\}$ and each of the logical symbols $=$, $\neg$, $\wedge$, $\vee$ or $\rightarrow$, the **quantifiers** $\forall$ or $\exists$ or the parentheses (or) is not a constant, function symbol or relation symbol from an FOL signature. Since I am only concerned with FOL signatures with unary and binary relation symbols, but without constants or function symbols, I use a convenient abbreviation as shorthand for this subclass of signatures. For each FOL signature $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar}\rangle$, for each $\mathcal{S}$, for each $\mathcal{A}$, I write

$\langle \mathcal{S}, \mathcal{A}\rangle$ for Σ
iff $\mathcal{C} = \emptyset$,
$\mathcal{F} = \emptyset$,
$\mathcal{R} = \mathcal{S} \cup \mathcal{A}$,
for each $s \in \mathcal{S}$: $\mathrm{Ar}(s) = 1$ and
for each $a \in \mathcal{A}$: $\mathrm{Ar}(a) = 2$.

Definition 6 *For each FOL signature* $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar}\rangle$*:*
Σ *is* **finite** *iff* $\mathcal{C}$*,* $\mathcal{F}$ *and* $\mathcal{R}$ *are finite.*

An interpretation of an FOL signature comprises a universe and an interpretation function, where the universe comprises a nonempty set of objects and the interpretation function assigns each constant, function symbol and relation symbol from the signature a meaning in terms of those objects. The meanings assigned by the interpretation function accord with the conditions encoded by the arity function of the signature.

Definition 7 *For each FOL signature* $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar}\rangle$*:*
Pair $\langle U, D\rangle$ *is an* **FOL interpretation** *of* Σ
iff U *is a nonempty set,*
D *is a total function with domain* $\mathcal{C} \cup \mathcal{F} \cup \mathcal{R}$*,*
for each $c \in \mathcal{C}$*:* $D(c) \in U$*,*
for each $f \in \mathcal{F}$*:* $D(f)$ *is a total function from* $U^{\mathrm{Ar}(f)}$ *to* U *and*
for each $r \in \mathcal{R}$*:* $D(r) \subseteq U^{\mathrm{Ar}(r)}$*.*

If Σ is an FOL signature and $I = \langle U, D\rangle$ is an FOL interpretation of Σ then I call U the **universe** of I, each member of U an **object** from I and D the **FOL interpretation function** of I. Since I am only concerned with FOL interpretations of FOL signatures with unary and binary relation symbols, but without constants or function symbols, I use a convenient abbreviation as shorthand for this subclass of interpretations. For each FOL signature $\langle \mathcal{S}, \mathcal{A}\rangle$, for each FOL interpretation $I = \langle U, D\rangle$ of $\langle \mathcal{S}, \mathcal{A}\rangle$, for each S, for each A, I write

$\langle U, S, A\rangle$ for $\langle U, D\rangle$
iff S is a total function from $\mathcal{S}$ to $\mathrm{Pow}(U)$[3],
A is a total function from $\mathcal{A}$ to $\mathrm{Pow}(U \times U)$ and
$D = S \cup A$.

A variable is a member of a fixed and countably infinite set of symbols and the variable names an object from some interpretation.

Definition 8 $\mathit{Var} = \{x_i | i \in \mathbb{N}\}$.

I call each member of Var a **variable**.

[3] For each set X, I write $\mathrm{Pow}(X)$ for the set of all subsets of X.

Definition 9 *For each FOL signature Σ, for each FOL interpretation $I = \langle U, D \rangle$ of Σ:*

β is an **assignment** *in I iff β is a total function from Var to U.*

If β is an assignment, I occasionally write β **assigns** u to x for $\beta(x) = u$. There exists a standard notation for an assignment which agrees on all but certain explicitly specified valuations with some other assignment.

Definition 10 *For each FOL signature Σ, for each FOL interpretation $I = \langle U, D \rangle$ of Σ, for each assignment β in I, for each $x_i \in Var$, for each $x_j \in Var$, for each $u \in U$:*

$$\beta_{[x_j \mapsto u]}(x_i) = \begin{cases} \beta(x_i) & \text{if } x_i \neq x_j \\ u & \text{if } x_i = x_j \end{cases}.$$

Clearly, for each FOL interpretation I, for each object u from I, for each variable x, for each assignment β in I, $\beta_{[x \mapsto u]}$ is an assignment in I.

A term is a finite and syntactically well-formed string of variables, constants and function symbols and each term denotes an object.

Definition 11 *For each FOL signature $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$:*

$\mathcal{T}_\Sigma$ is the smallest set such that

for each $x \in Var$: $x \in \mathcal{T}_\Sigma$,

for each $c \in \mathcal{C}$: $c \in \mathcal{T}_\Sigma$ and

for each $f \in \mathcal{F}$, for each $t_1 \in \mathcal{T}_\Sigma$, ..., for each $t_{\mathrm{Ar}(f)} \in \mathcal{T}_\Sigma$: $f(t_1, \ldots, t_{\mathrm{Ar}(f)}) \in \mathcal{T}_\Sigma$.

Definition 12 *For each FOL signature $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$, for each FOL interpretation $I = \langle U, D \rangle$ of Σ, for each assignment β in I:*

$[]_I^\beta$ is the total function from $\mathcal{T}_\Sigma$ to U such that

for each $x \in Var$: $[x]_I^\beta = \beta(x)$,

for each $c \in \mathcal{C}$: $[c]_I^\beta = D(c)$ and

for each $f \in \mathcal{F}$, for each $t_1 \in \mathcal{T}_\Sigma$, ..., for each $t_{\mathrm{Ar}(f)} \in \mathcal{T}_\Sigma$:

$$[f(t_1, \ldots, t_{\mathrm{Ar}(f)})]_I^\beta = D(f)\langle [t_1]_I^\beta, \ldots, [t_{\mathrm{Ar}(f)}]_I^\beta \rangle.$$

If Σ is an FOL signature and I is an FOL interpretation of Σ then I call each member of $\mathcal{T}_\Sigma$ an **FOL term** from Σ and $[]_I^\beta$ the **FOL term interpretation function** under β in I.

A formula is a finite and syntactically well-formed string of FOL terms, relation symbols, logical symbols, quantifiers and parentheses.

Definition 13 *For each FOL signature $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$:*

$\mathcal{F}_\Sigma$ is the smallest set such that

for each $t_1 \in \mathcal{T}_\Sigma$, for each $t_2 \in \mathcal{T}_\Sigma$: $(t_1 = t_2) \in \mathcal{F}_\Sigma$,

for each $r \in \mathcal{R}$, for each $t_1 \in \mathcal{T}_\Sigma$, ..., for each $t_{\mathrm{Ar}(r)} \in \mathcal{T}_\Sigma$: $r(t_1, \ldots, t_{\mathrm{Ar}(r)}) \in \mathcal{F}_\Sigma$,

for each $f \in \mathcal{F}_\Sigma$: $\neg f \in \mathcal{F}_\Sigma$,

for each $f_1 \in \mathcal{F}_\Sigma$ for each $f_2 \in \mathcal{F}_\Sigma$: $(f_1 \wedge f_2) \in \mathcal{F}_\Sigma$,

for each $f_1 \in \mathcal{F}_\Sigma$ for each $f_2 \in \mathcal{F}_\Sigma$: $(f_1 \vee f_2) \in \mathcal{F}_\Sigma$,

for each $f_1 \in \mathcal{F}_\Sigma$ *for each* $f_2 \in \mathcal{F}_\Sigma$: $(f_1 \rightarrow f_2) \in \mathcal{F}_\Sigma$,
for each $x \in \mathit{Var}$, *for each* $f \in \mathcal{F}_\Sigma$: $(\forall x f) \in \mathcal{F}_\Sigma$ *and*
for each $x \in \mathit{Var}$, *for each* $f \in \mathcal{F}_\Sigma$: $(\exists x f) \in \mathcal{F}_\Sigma$.

If Σ is an FOL signature then I call each member of $\mathcal{F}_\Sigma$ an **FOL formula** from Σ. For notational ease, I use a number of convenient abbreviations as shorthand for genuine formulae. For each FOL signature Σ, for each $n \in \mathbb{N}$, for each $f_1 \in \mathcal{F}_\Sigma$, for each $f_2 \in \mathcal{F}_\Sigma$, ..., for each $f_{n+3} \in \mathcal{F}_\Sigma$, for each quantifier $\diamondsuit_1$, ..., for each quantifier $\diamondsuit_{n+2}$, for each $x_1 \in \mathit{Var}$, ..., for each $x_{n+2} \in \mathit{Var}$, I write
$\bigvee\emptyset$ for $\neg(x = x)$,
$\bigvee\{f_1, f_2, \ldots, f_{n+1}\}$ for $(f_1 \vee \bigvee\{f_2, \ldots, f_{n+1}\})$,
$(f_1 \wedge f_2 \wedge \cdots \wedge f_{n+3})$ for $(f_1 \wedge (f_2 \wedge \cdots \wedge f_{n+3}))$,
$(\diamondsuit_1 x_1 \diamondsuit_2 x_2 \cdots \diamondsuit_{n+2} x_{n+2} f_1)$ for $(\diamondsuit_1 x_1 (\diamondsuit_2 x_2 \cdots \diamondsuit_{n+2} x_{n+2} f_1))$.

There is a well-known notion of free variable.

Definition 14 *For each FOL signature* $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$:
FV_Σ *is the total function from* $\mathcal{T}_\Sigma \cup \mathcal{F}_\Sigma$ *to* $\mathrm{Pow}(\mathit{Var})$ *such that*
for each $x \in \mathit{Var}$: $\mathrm{FV}_\Sigma(x) = \{x\}$,
for each $c \in \mathcal{C}$: $\mathrm{FV}_\Sigma(c) = \emptyset$,
for each $f \in \mathcal{F}$, *for each* $t_1 \in \mathcal{T}_\Sigma$, ..., *for each* $t_{\mathrm{Ar}(f)} \in \mathcal{T}_\Sigma$:
$\mathrm{FV}_\Sigma(f(t_1, \ldots, t_{\mathrm{Ar}(f)})) = \mathrm{FV}_\Sigma(t_1) \cup \ldots \cup \mathrm{FV}_\Sigma(t_{\mathrm{Ar}(f)})$,
for each $t_1 \in \mathcal{T}_\Sigma$, *for each* $t_2 \in \mathcal{T}_\Sigma$: $\mathrm{FV}_\Sigma((t_1 = t_2)) = \mathrm{FV}_\Sigma(t_1) \cup \mathrm{FV}_\Sigma(t_2)$,
for each $r \in \mathcal{R}$, *for each* $t_1 \in \mathcal{T}_\Sigma$, ..., *for each* $t_{\mathrm{Ar}(r)} \in \mathcal{T}_\Sigma$:
$\mathrm{FV}_\Sigma(r(t_1, \ldots, t_{\mathrm{Ar}(r)})) = \mathrm{FV}_\Sigma(t_1) \cup \ldots \cup \mathrm{FV}_\Sigma(t_{\mathrm{Ar}(r)})$,
for each $f \in \mathcal{F}_\Sigma$: $\mathrm{FV}_\Sigma(\neg f) = \mathrm{FV}_\Sigma(f)$,
for each $f_1 \in \mathcal{F}_\Sigma$ *for each* $f_2 \in \mathcal{F}_\Sigma$:
$\mathrm{FV}_\Sigma((f_1 \wedge f_2)) = \mathrm{FV}_\Sigma((f_1 \vee f_2)) = \mathrm{FV}_\Sigma((f_1 \rightarrow f_2)) = \mathrm{FV}_\Sigma(f_1) \cup \mathrm{FV}_\Sigma(f_2)$ *and*
for each $x \in \mathit{Var}$, *for each* $f \in \mathcal{F}_\Sigma$: $\mathrm{FV}_\Sigma((\forall x f)) = \mathrm{FV}_\Sigma((\exists x f)) = \mathrm{FV}_\Sigma(f) \setminus \{x\}$.

If Σ is an FOL signature and f is an FOL formula from Σ then I call each member of $\mathrm{FV}_\Sigma(f)$ a **free variable** in f.

A sentence is an FOL formula with no free variables.

Definition 15 *For each FOL signature* $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$, *for each* $f \in \mathcal{F}_\Sigma$:
f *is an* **FOL sentence** *iff* $\mathrm{FV}_\Sigma(f) = \emptyset$.

An assignment FOL satisfies an FOL formula in an FOL interpretation iff the formula is true when interpreted by the interpretation and assignment.

Definition 16 *For each FOL signature* $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \mathrm{Ar} \rangle$, *for each FOL interpretation* $I = \langle U, D \rangle$ *of* Σ, *for each assignment* β *in* I:
for each $t_1 \in \mathcal{T}_\Sigma$, *for each* $t_2 \in \mathcal{T}_\Sigma$: $I \models_\beta (t_1 = t_2)$ *iff* $[t_1]_I^\beta = [t_2]_I^\beta$,
for each $r \in \mathcal{R}$, *for each* $t_1 \in \mathcal{T}_\Sigma$, ..., *for each* $t_{\mathrm{Ar}(r)} \in \mathcal{T}_\Sigma$:
$I \models_\beta r(t_1, \ldots, t_{\mathrm{Ar}(r)})$ *iff* $\langle [t_1]_I^\beta, \ldots, [t_{\mathrm{Ar}(r)}]_I^\beta \rangle \in D(r)$,

for each $f \in \mathcal{F}_\Sigma$*:* $I \models_\beta \neg f$ *iff not* $I \models_\beta f$*,*

for each $f_1 \in \mathcal{F}_\Sigma$ *for each* $f_2 \in \mathcal{F}_\Sigma$*:* $I \models_\beta (f_1 \wedge f_2)$ *iff* $I \models_\beta f_1$ *and* $I \models_\beta f_2$*,*

for each $f_1 \in \mathcal{F}_\Sigma$ *for each* $f_2 \in \mathcal{F}_\Sigma$*:* $I \models_\beta (f_1 \vee f_2)$ *iff* $I \models_\beta f_1$ *or* $I \models_\beta f_2$ *or both,*

for each $f_1 \in \mathcal{F}_\Sigma$ *for each* $f_2 \in \mathcal{F}_\Sigma$*:* $I \models_\beta (f_1 \rightarrow f_2)$ *iff not* $I \models_\beta f_1$ *or* $I \models_\beta f_2$*,*

for each $x \in Var$*, for each* $f \in \mathcal{F}_\Sigma$*:* $I \models_\beta (\forall x f)$ *iff for each* $u \in U$*:* $I \models_{\beta_{[x \mapsto u]}} f$ *and*

for each $x \in Var$*, for each* $f \in \mathcal{F}_\Sigma$*:* $I \models_\beta (\exists x f)$ *iff for some* $u \in U$*:* $I \models_{\beta_{[x \mapsto u]}} f$*.*

If Σ is an FOL signature, I is an FOL interpretation of Σ, f is an FOL formula from Σ and β is an assignment in I, I occasionally write β **FOL satisfies** f **in** I for $I \models_\beta f$.

An FOL interpretation FOL satisfies an FOL formula iff each assignment in the interpretation FOL satisfies the formula in the interpretation.

Definition 17 *For each FOL signature* Σ*, for each FOL interpretation* I *of* Σ*, for each* $f \in \mathcal{F}_\Sigma$*:*

$I \models f$ *iff for each assignment* β *in* I*:* $I \models_\beta f$*.*

If Σ is an FOL signature, I is an FOL interpretation of Σ and f is an FOL formula from Σ, I occasionally write I **FOL satisfies** f for $I \models f$.

An FOL theory is a set — possibly empty or infinite — of FOL formulae and an FOL interpretation FOL satisfies the theory iff the interpretation FOL satisfies each member of the theory.

Definition 18 *For each FOL signature* Σ*, for each interpretation* I *of* Σ*, for each* $\theta \subseteq \mathcal{F}_\Sigma$*:*

$I \models \theta$ *iff for each* $f \in \theta$*:* $I \models f$*.*

If Σ is an FOL signature and I is an FOL interpretation of Σ then I call each subset θ of $\mathcal{F}_\Sigma$ an **FOL theory** from Σ and occasionally write I **FOL satisfies** θ for $I \models \theta$. I call an FOL theory **sentential** iff each of its members is a sentence.

An FOL theory or FOL formula is FOL satisfiable iff some FOL interpretation FOL satisfies the theory or formula.

Definition 19 *For each FOL signature* Σ*, for each* $\theta \subseteq \mathcal{F}_\Sigma$*, for each* $f \in \mathcal{F}_\Sigma$*:*

θ *is* **FOL satisfiable** *iff for some FOL interpretation* I *of* Σ*:* $I \models \theta$ *and*

f *is* **FOL satisfiable** *iff for some FOL interpretation* I *of* Σ*:* $I \models f$*.*

An FOL theory FOL entails an FOL formula iff each FOL interpretation that satisfies the theory satisfies the formula.

Definition 20 *For each FOL signature* Σ*, for each* $\theta \subseteq \mathcal{F}_\Sigma$*, for each* $f \in \mathcal{F}_\Sigma$*:*

$\theta \models f$ *iff for each FOL interpretation* I *of* Σ*: if* $I \models \theta$ *then* $I \models f$*.*

If Σ is an FOL signature, θ is an FOL theory from Σ and f is an FOL formula from Σ, I occasionally write θ **FOL entails** f for $\theta \models f$.

It is a standard result that the union of a sentential theory and a singleton sentential theory is not FOL satisfiable iff the sentential theory entails the negation of the unique member of the singleton theory.

Proposition 3 *For each FOL signature Σ, for each sentential $\theta \subseteq \mathcal{F}_\Sigma$, for each sentence $f \in \mathcal{F}_\Sigma$:*
$\theta \cup \{f\}$ *is not FOL satisfiable iff* $\theta \models \neg f$.

The FOL language and its notions are notational variants of the language and notions of [Bar77, pages 17–22]. For each FOL signature with countably many symbols, [Bar77, pages 22–37] provides a calculus $\mathfrak{C}$ which generates an inference relation $\vdash$ between sentential theories and sentences and proves that the calculus is sound and complete with respect to FOL entailment.

Proposition 4 *For each FOL signature Σ, for each sentential $\theta \subseteq \mathcal{F}_\Sigma$, for each sentence $f \in \mathcal{F}_\Sigma$:*
$\theta \models f$ *iff* $\theta \vdash f$.

If θ is an FOL theory and f is an FOL sentence, I occasionally write θ **infers** f for $\theta \vdash f$.

2.3 Speciate Re-entrant Logic

SRL is specifically designed to provide a formalism for HPSG. Yet, it conflicts with [PS94]'s view that the interpretation of a mathematical theory about phenomena does perforce require mathematical structures to model phenomena (refer to the citation on page 6). Rather, it is based on the assumption that the authors of an HPSG theory can indicate the correspondence between the phenomena of their intended interpretation and the mathematical structures that they posit to model those phenomena no better than they can indicate the interpretation itself. Thus, SRL is based on a direct relationship between theories and linguistic objects rather than on an indirect relationship via feature structures that stand in a one-to-one correspondence with linguistic objects. However, this methodological difference, as discussed on page 3, implies no observational difference between the two approaches.

[PS94]'s demands on (i) "being explicit as to what sorts of constructs are assumed (i.e. what ontological categories of linguistic objects we suppose to populate the empirical domain)" ([PS94, page 7]), (ii) "being mathematically rigorous as to what structures are used to model them" ([PS94, page 7]) and the requirement of (iii) a one-to-one correspondence between the modelling structures and linguistic objects (possibly contained in (ii)) collapse into (i) under SRL's direct approach. SRL fulfills demand (i) by providing a class of signatures that, informally speaking, characterize the domain of linguistic objects that are 'in principle' considered to be possible. Next, SRL fulfills the requirement that "the theory itself actually count as a theory in the technical sense of precisely characterizing those modelling structures [now: linguistic objects, B.A.] that are regarded as admissible or well-formed" ([PS94, page 7]) by interpreting a theory as the set of linguistic objects of the domain that the theory is true of.

An SRL signature comprises two sets of symbols and a function that encodes certain conditions on the interpretation of the symbols.

Definition 21 *Triple* $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$ *is an* **SRL signature**
iff $\mathcal{S}$ *is a set,*
$\mathcal{A}$ *is a set and*
$\mathcal{F}$ *is a total function from* $\mathcal{S} \times \mathcal{A}$ *to* $\mathrm{Pow}(\mathcal{S})$.

If $\Sigma = \langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$ is an SRL signature then I call each member of $\mathcal{S}$ a **species** from Σ, each member of $\mathcal{A}$ an **attribute** from Σ and $\mathcal{F}$ the **appropriateness function** of Σ. I occasionally write a is **appropriate for** s for $\mathcal{F}\langle s, a \rangle \neq \emptyset$ and a **deserves** s' **for** s for $s' \in \mathcal{F}\langle s, a \rangle$. For notational facility, I assume that each of the symbols :, $\sim$, $\approx$, $\neg$, $\wedge$, $\vee$, $\rightarrow$, (or) is not a species or an attribute from an SRL signature.

Definition 22 *For each SRL signature* $\Sigma = \langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$*:*

Σ *is* **limited** *iff* $\mathcal{S}$ *is finite and*

Σ *is* **finite** *iff* $\mathcal{S}$ *is finite and* $\mathcal{A}$ *is finite.*

An SRL interpretation of an SRL signature comprises a universe and two interpretation functions, where the universe comprises a set of objects and the interpretation functions assign each species and each attribute in the signature a meaning in terms of those objects. The meanings assigned by the interpretation functions accord with the conditions encoded by the appropriateness function of the signature.

Definition 23 *For each SRL signature* $\Sigma = \langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$*:*

Triple $\langle U, S, A \rangle$ *is an* **SRL interpretation** *of* Σ

iff U *is a set,*

S *is a total function from* U *to* $\mathcal{S}$*,*

A *is a total function from* $\mathcal{A}$ *to the set of partial functions from* U *to* U *and*

for each $a \in \mathcal{A}$*, for each* $u \in U$*:*

$A(a)(u)$ *is defined iff* $\mathcal{F}\langle S(u), a \rangle \neq \emptyset$ *and*

if $A(a)(u)$ *is defined then* $S(A(a)(u)) \in \mathcal{F}\langle S(u), a \rangle$*.*

If Σ is an SRL signature and $I = \langle U, S, A \rangle$ is an SRL interpretation of Σ then I call U the **universe** of I, each member of U an **object** from I, S the **species interpretation function** of I, and A the **attribute interpretation function** of I. Intuitively, each species names one of a set of disjoint subsets of the universe and the species interpretation function assigns each object the name of the unique subset to which the object belongs; each attribute names one of a set of partial functions from the universe to the universe and the attribute interpretation function assigns each attribute the partial function it names; and the appropriateness function encodes — and the last condition in the definition of an interpretation enforces — a relationship between the interpretations of species and attributes: If $\mathcal{F}\langle s, a \rangle = \emptyset$ then the partial function that attribute a names acts upon no object in the subset that species s names, and if $\mathcal{F}\langle s, a \rangle \neq \emptyset$ then the partial function that attribute a names acts upon each object in the subset that species s names to yield an object in a subset named by one of the species in $\mathcal{F}\langle s, a \rangle$.

Definition 24 *For each SRL signature* Σ*, for each SRL interpretation* $I = \langle U, S, A \rangle$ *of* Σ*:*

I *is* **nonempty** *iff* $U \neq \emptyset$*.*

An SRL term is a : symbol followed by a finite string of attributes and the term names the functional composition of the partial functions that the attributes name.

Definition 25 *For each SRL signature* $\Sigma = \langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$*:*

$\mathcal{T}_\Sigma$ *is the smallest set such that*

$: \in \mathcal{T}_\Sigma$ *and*

for each $t \in \mathcal{T}_\Sigma$*, for each* $a \in \mathcal{A}$*:* $ta \in \mathcal{T}_\Sigma$*.*

Definition 26 *For each SRL signature $\Sigma = \langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$, for each SRL interpretation $I = \langle U, S, A \rangle$ of Σ:*

$[]_I$ *is the total function from $\mathcal{T}_\Sigma$ to the set of partial functions from U to U such that*

$[:]_I$ *is the identity function from U to U and*

for each $t \in \mathcal{T}_\Sigma$, for each $a \in \mathcal{A}$, for each $u \in U$:

$[ta]_I(u)$ *is defined iff* $[t]_I(u)$ *and* $A(a)([t]_I(u))$ *are defined and*

if $[ta]_I(u)$ *is defined then* $[ta]_I(u) = A(a)([t]_I(u))$.

If Σ is an SRL signature and I is an SRL interpretation of Σ then I call each member of $\mathcal{T}_\Sigma$ an **SRL term** from Σ and $[]_I$ the **SRL term interpretation function** under I.

An SRL description is a finite and syntactically well-formed string of SRL terms, species and logical symbols and the description denotes the set of objects that the description is true of.

Definition 27 *For each SRL signature $\Sigma = \langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$:*

$\mathcal{D}_\Sigma$ *is the smallest set such that*

for each $t \in \mathcal{T}_\Sigma$, for each $s \in \mathcal{S}$: $t \sim s \in \mathcal{D}_\Sigma$,

for each $t_1 \in \mathcal{T}_\Sigma$, for each $t_2 \in \mathcal{T}_\Sigma$: $t_1 \approx t_2 \in \mathcal{D}_\Sigma$,

for each $d \in \mathcal{D}_\Sigma$: $\neg d \in \mathcal{D}_\Sigma$,

for each $d_1 \in \mathcal{D}_\Sigma$, for each $d_2 \in \mathcal{D}_\Sigma$: $(d_1 \wedge d_2) \in \mathcal{D}_\Sigma$,

for each $d_1 \in \mathcal{D}_\Sigma$, for each $d_2 \in \mathcal{D}_\Sigma$: $(d_1 \vee d_2) \in \mathcal{D}_\Sigma$ and

for each $d_1 \in \mathcal{D}_\Sigma$, for each $d_2 \in \mathcal{D}_\Sigma$: $(d_1 \rightarrow d_2) \in \mathcal{D}_\Sigma$.

Definition 28 *For each SRL signature $\Sigma = \langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$, for each SRL interpretation $I = \langle U, S, A \rangle$ of Σ:*

$[\![]\!]_I$ *is the total function from $\mathcal{D}_\Sigma$ to* $\mathrm{Pow}(U)$ *such that*

for each $t \in \mathcal{T}_\Sigma$, for each $s \in \mathcal{S}$:

$$[\![t \sim s]\!]_I = \left\{ u \in U \;\middle|\; \begin{array}{l} [t]_I(u) \textit{ is defined and} \\ S([t]_I(u)) = s \end{array} \right\},$$

for each $t_1 \in \mathcal{T}_\Sigma$, for each $t_2 \in \mathcal{T}_\Sigma$:

$$[\![t_1 \approx t_2]\!]_I = \left\{ u \in U \;\middle|\; \begin{array}{l} [t_1]_I(u) \textit{ is defined,} \\ [t_2]_I(u) \textit{ is defined and} \\ [t_1]_I(u) = [t_2]_I(u) \end{array} \right\},$$

for each $d \in \mathcal{D}_\Sigma$: $[\![\neg d]\!]_I = U \setminus [\![d]\!]_I$,

for each $d_1 \in \mathcal{D}_\Sigma$, for each $d_2 \in \mathcal{D}_\Sigma$: $[\![(d_1 \wedge d_2)]\!]_I = [\![d_1]\!]_I \cap [\![d_1]\!]_I$,

for each $d_1 \in \mathcal{D}_\Sigma$, for each $d_2 \in \mathcal{D}_\Sigma$: $[\![(d_1 \vee d_2)]\!]_I = [\![d_1]\!]_I \cup [\![d_1]\!]_I$ and

for each $d_1 \in \mathcal{D}_\Sigma$, for each $d_2 \in \mathcal{D}_\Sigma$: $[\![(d_1 \rightarrow d_2)]\!]_I = (U \setminus [\![d_1]\!]_I) \cup [\![d_2]\!]_I$.

If Σ is an SRL signature and I is an SRL interpretation of Σ then I call each member of $\mathcal{D}_\Sigma$ an **SRL description** from Σ and $[\![]\!]_I$ the **SRL description denotation function** under I. Intuitively, description $t \sim s$ is true of object u iff

the partial function term t names maps u to object u', and u' is in the subset species s names;

description $t_1 \approx t_2$ is true of object u iff

the partial function term t_1 names maps u to object u_1, the partial function term t_2 names maps u to object u_2, and u_1 and u_2 are the same object;

description $\neg d$ is true of object u iff d is false of u; description $(d_1 \wedge d_2)$ is true of object u iff d_1 is true of u and d_2 is true of u; description $(d_1 \vee d_2)$ is true of object u iff d_1 is true of u or d_2 is true of u; and description $(d_1 \rightarrow d_2)$ is true of object u iff if d_1 is true of u then d_2 is true of u.

An SRL theory is a set — possibly empty or infinite — of SRL descriptions, and an SRL theory is true of an object iff each description in the theory is true of the object.

Definition 29 *For each SRL signature Σ, for each SRL interpretation $I = \langle U, S, A \rangle$ of Σ:*

$[\![\![\,]\!]\!]_I$ *is the total function from* $\mathrm{Pow}(\mathcal{D}_\Sigma)$ *to* $\mathrm{Pow}(U)$ *such that*

for each $\theta \subseteq \mathcal{D}_\Sigma$: $[\![\![\theta]\!]\!]_I = \{u \in U | \text{for each } d \in \theta\text{: } u \in [\![d]\!]_I\}$.

If Σ is an SRL signature and I is an SRL interpretation of Σ then I call each subset of $\mathcal{D}_\Sigma$ an **SRL theory** from Σ and $[\![\![\,]\!]\!]_I$ the **SRL theory denotation function** under I.

An SRL interpretation satisfies an SRL theory or SRL description iff the theory or description is true of some object in the interpretation.

Definition 30 *For each SRL signature Σ, for each SRL interpretation I of Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:*

I **SRL satisfies** *θ under Σ iff $[\![\![\theta]\!]\!]_I \neq \emptyset$ and*

I **SRL satisfies** *d under Σ iff $[\![d]\!]_I \neq \emptyset$.*

Notice that each nonempty SRL interpretation SRL satisfies description $:\approx:$.

Proposition 5 *For each SRL signature Σ, for each nonempty SRL interpretation I of Σ:*

I SRL satisfies $:\approx:$ under Σ.

An SRL theory or SRL description is satisfiable iff some SRL interpretation satisfies the theory or description.

Definition 31 *For each SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:*

θ is **SRL satisfiable** *under Σ*

iff for some SRL interpretation I of Σ: I satisfies θ under Σ and

d is **SRL satisfiable** *under Σ*

iff for some SRL interpretation I of Σ: I SRL satisfies d under Σ.

An SRL interpretation models an SRL theory or SRL description iff the interpretation is nonempty and the theory or description is true of each object in the interpretation.

Definition 32 *For each SRL signature Σ, for each SRL interpretation $I = \langle U, S, A \rangle$ of Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:*

I **SRL models** *θ under Σ iff I is nonempty and $[\![\![\theta]\!]\!]_I = U$ and*

I **SRL models** *d under Σ iff I is nonempty and $[\![d]\!]_I = U$.*

Notice that each nonempty SRL interpretation SRL models the empty theory.

Proposition 6 *For each SRL signature Σ, for each nonempty SRL interpretation I of Σ:*

I SRL models $\emptyset$ under Σ.

An SRL theory or SRL description is modellable iff some SRL interpretation models the theory or description.

Definition 33 *For each SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$:*

θ is **SRL modellable** *under Σ*

iff for some SRL interpretation I of Σ: I SRL models θ under Σ and

d is **SRL modellable** *under Σ*

iff for some SRL interpretation I of Σ: I SRL models d under Σ.

[KSA, page 15] proves that modellability in some finite SRL signature is Π^0_1-hard.

Proposition 7 *For some finite SRL signature Σ:*

$\{d \in \mathcal{D}_\Sigma | d$ *is SRL modellable under* $\Sigma\}$ *is* Π^0_1*-hard.*

Under the assumption that a natural language can be construed as an SRL interpretation, SRL defines what it is for an SRL theory to be true of an object in the language: The theory is true of the object in the language iff the denotation of the theory contains the object when interpreted by the language. However, the linguistically relevant question is rather what it is for a theory to be true of a natural language, a collection of objects. Yet, SRL fails to formally capture the HPSGian's intuitive notion of linguistic truth, and thus fails to provide a means to falsify an SRL theory with respect to a natural language.

[Kin] cautiously interprets the statement that

> The role of the linguistic theory is to give a precise specification of which feature structure are to be considered admissible; the types of linguistic entities that correspond to the admissible feature structures constitute the predictions of the theory. [PS94, page 8]

as a necessary condition of linguistic truth in HPSG and claims to capture this intuition in formal SRL terms by means of so-called exhaustive models (which play no immediate role in the present paper).

Thesis 1 *For each SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each natural language L:*

if θ is true of L

then for some SRL interpretation I of Σ:

L can be construed as I and

I is an exhaustive model of θ.

[Kin] then defines the following relation between SRL theories and SRL descriptions. An SRL theory predicts an SRL description iff some SRL interpretation models the theory and satisfies the description.

Definition 34 *For each SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:*
θ **SRL predicts** *d under Σ*
iff for some SRL interpretation I of Σ:
I SRL models θ under Σ and I SRL satisfies d under Σ.

If Σ is an SRL signature, θ is an SRL theory from Σ and d is an SRL description from Σ then I call d a **prediction** of θ iff θ SRL predicts d under Σ. Informally, each prediction of an SRL theory is interpreted as asserting the *existence* of some object the prediction is true of.

Notice that there exists a strong relationship between SRL satisfiability and SRL prediction as well as SRL modellability and SRL prediction.

Proposition 8 *For each SRL signature Σ, for each $d \in \mathcal{D}_\Sigma$:*
d is SRL satisfiable under Σ iff $\emptyset$ SRL predicts d under Σ.

Proof For each SRL signature Σ, for each $d \in \mathcal{D}_\Sigma$:
d is SRL satisfiable under Σ
$\Longleftrightarrow$ for some nonempty SRL interpretation I of Σ:
I SRL satisfies d under Σ
$\Longleftrightarrow$ for some SRL interpretation I of Σ:
I SRL models $\emptyset$ under Σ and I SRL satisfies d under Σ by Proposition 6
$\Longleftrightarrow$ $\emptyset$ SRL predicts d under Σ.
□

Proposition 9 *For each SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$:*
d is SRL modellable under Σ iff θ SRL predicts $:\approx:$ under Σ.

Proof For each SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$:
θ is SRL modellable under Σ
$\Longleftrightarrow$ for some nonempty SRL interpretation I of Σ:
I SRL models θ under Σ
$\Longleftrightarrow$ for some nonempty SRL interpretation I of Σ:
I SRL models θ under Σ and I SRL satisfies $:\approx:$ under Σ by Proposition 5
$\Longleftrightarrow$ for some SRL interpretation I of Σ:
I SRL models θ under Σ and I SRL satisfies $:\approx:$ under Σ
$\Longleftrightarrow$ θ SRL predicts $:\approx:$ under Σ.
□

[Kin] goes on to prove the following.

Proposition 10 *For each SRL signature Σ, for each SRL interpretation I of Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$:*
if I is an exhaustive model of θ
then for each $d \in \mathcal{D}_\Sigma$:
θ SRL predicts d under Σ iff I SRL satisfies d under Σ.

Consequently, from Thesis 1 and Proposition 10, [Kin] arrives at the following thesis.

Thesis 2 *For each SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each natural language L:*
if θ is true of L
then for some interpretation I of Σ:
L can be construed as I and
for each $d \in \mathcal{D}_\Sigma$:
θ SRL predicts d under Σ iff I SRL satisfies d under Σ.

More detailed, for each theory, for each description and for each natural language, if the theory is true of the language and the theory predicts the description then the language contains some object the description is true of, and if the theory is true of the language and the language contains an object the description is true of then the theory predicts the description.

Notice two things related to the testability of SRL theories with respect to natural languages. Firstly, since Thesis 2 provides only a necessary, but not sufficient condition of linguistic truth, it only allows the *falsification* of SRL theories with respect to natural languages. Secondly, only those predictions of a theory with experimentally verifiable or falsifiable content allow the falsification of the theory with respect to a natural language. If, for example, the theory predicts the existence of an object with a specific phrase structure component, the prediction, hence the theory, cannot be experimentally falsified with respect to English by the experimental means at present available to linguistics. By contrast, if the theory predicts the existence of an object with the phonological component *Bikes are a sunshine thing*, the prediction, hence the theory, can be falsified with respect to German, since speakers of German judge this phonology as ill-formed in German. Yet, since the verification of a theory with respect to a natural language is physically impossible, a prediction relation that enables the falsification of a theory is sufficient for a deductive science.

Chapter 3

Correlation

In this chapter, I establish a correlation between SRL non-prediction and FOL inference. The chapter comprises two sections. In the first section, I assign each limited SRL signature, nonempty SRL interpretation and SRL description an FOL analog. In the second section, I use the correlations between the SRL notions and their FOL analogs to show that an SRL theory does not predict an SRL description if and only if some FOL analog of the theory infers some FOL analog of the description.

3.1 Translation

Signatures

I convert each limited SRL signature into a corresponding FOL signature and the SRL signature's restrictions on its SRL interpretations into a corresponding FOL theory. I consider each SRL species to be an FOL unary relation symbol and each SRL attribute to be an FOL binary relation symbol. Thus, for each SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$, $\langle \mathcal{S}, \mathcal{A} \rangle$ is the corresponding FOL signature. However, in two respects, an SRL signature imposes more restrictions on an SRL interpretation than an FOL signature imposes on an FOL interpretation: In an SRL interpretation, each object is required to be of a unique species, whereas an object in an FOL interpretation need not be in the denotation of a unique unary relation symbol. Secondly, an SRL interpretation must interpret each attribute as a partial function which obeys the appropriateness conditions, but no such requirement applies to an FOL interpretation when interpreting each binary relation symbol. I call the former the **s-restriction**, the latter the **a-restriction**. Since an FOL signature, unlike an SRL signature, does not impose these restrictions on its interpretations, I will carry them over by different means: By translating them into an FOL theory.

Definition 35 *For each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$:*

$$\begin{aligned}
&\mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle) \\
&= \{(\forall x \bigvee \{s(x) | s \in \mathcal{S}\})\} \\
&\quad \cup \{(\forall x \neg (s_1(x) \wedge s_2(x))) | s_1 \in \mathcal{S}, s_2 \in \mathcal{S}, s_1 \neq s_2\} \\
&\quad \cup \{(\forall x \forall y \forall z ((a(x,y) \wedge a(x,z)) \rightarrow (y = z))) | a \in \mathcal{A}\} \\
&\quad \cup \{(\forall x (s(x) \rightarrow \bigvee \{(\exists y (a(x,y) \wedge s'(y))) | s' \in \mathcal{F}\langle s, a \rangle\})) | s \in \mathcal{S}, a \in \mathcal{A}, \mathcal{F}\langle s, a \rangle \neq \emptyset\} \\
&\quad \cup \{(\forall x (s(x) \rightarrow \neg (\exists y\, a(x,y)))) | s \in \mathcal{S}, a \in \mathcal{A}, \mathcal{F}\langle s, a \rangle = \emptyset\}.
\end{aligned}$$

The first and second lines together encode the s-restriction, the fact that each object lies in a unique unary relation. The third, fourth and fifth lines express the a-restriction: Binary relations are partial-functional; if an attribute a is appropriate for a species s then each object in the unary s-relation must stand in the a-relation with some object in some unary s'-relation that a deserves for s; and no object in any unary relation s may stand in the a-relation with an object if attribute a is not appropriate for species s.

Thus, each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$ has the FOL signature $\langle \mathcal{S}, \mathcal{A} \rangle$ and the FOL theory $\mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle)$ as its FOL analog.

Interpretations

I convert each nonempty SRL interpretation into a corresponding FOL interpretation. Suppose that $\langle U, S, A \rangle$ is a nonempty SRL interpretation in an SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$. Firstly, I consider U to be the corresponding FOL universe. Secondly, since I view each SRL species as an FOL unary relation symbol, I need an interpretation function that assigns each unary relation symbol from $\langle \mathcal{S}, \mathcal{A} \rangle$ a set of objects from U such that an object is in the denotation of a relation symbol s iff the species interpretation function S assigns species s to that object. The function $\vec{\ }$, defined below, converts a species interpretation function S into just such a unary relation symbol interpretation function $\vec{S}$. In other words, $\vec{S}$ is the FOL way of expressing S.

Definition 36 *For each Y, for each Z, for each $R \subseteq Y \times Z$:*
$\vec{R}$ is the total function from Z to $\mathrm{Pow}(Y)$ such that
for each $z \in Z$: $\vec{R}(z) = \{y \in Y | \langle y, z \rangle \in R\}$.

Thirdly, I assign the attribute interpretation function A an FOL analog. A assigns each SRL attribute from $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$ a partial function from U to U. Extensionally, this function can be viewed as a binary relation, a set of pairs.[1] Thus, on the FOL side, I now view A as a function which interprets binary relation symbols in $\langle \mathcal{S}, \mathcal{A} \rangle$ by assigning them the binary relation on U that I view as a partial function from U to U on the SRL side.

Summing up, each nonempty SRL interpretation $\langle U, S, A \rangle$ of an SRL signature Σ has the FOL interpretation $\langle U, \vec{S}, A \rangle$ as its FOL analog.

The translation of a nonempty SRL interpretation of a limited SRL signature is an FOL interpretation that mimics the SRL interpretation in FOL: The FOL interpretation corresponding to the SRL interpretation obeys the s- and a-restrictions of the SRL signature by satisfying the translation of the restrictions encoded by the signature.

Proposition 11 *For each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$, for each triple $\langle U, S, A \rangle$:*
$\langle U, S, A \rangle$ is a nonempty SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$
iff $\vec{S}$ is defined,
$\langle U, \vec{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$ and
$\langle U, \vec{S}, A \rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle)$.

[1] The reader may prefer to say that the *graph* of a function is a set of pairs, not the function itself. I make no distinction between a function and its graph.

Proof For each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$, for each triple $\langle U, S, A \rangle$:
$\langle U, S, A \rangle$ is a nonempty SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$
$\iff$ U is a nonempty set,
S is a total function from U to $\mathcal{S}$,
A is a total function from $\mathcal{A}$ to the set of partial functions from U to U and
for each $a \in \mathcal{A}$, for each $u \in U$:
$A(a)(u)$ is defined iff $\mathcal{F}\langle S(u), a \rangle \neq \emptyset$ and
if $A(a)(u)$ is defined then $S(A(a)(u)) \in \mathcal{F}\langle S(u), a \rangle$
$\iff$ U is a nonempty set,
S is a total function from U to $\mathcal{S}$,
A is a total function from $\mathcal{A}$ to the set of partial functions from U to U,
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle \neq \emptyset$ then for each $u \in U$:
if $S(u) = s$ then for some $s' \in \mathcal{F}\langle s, a \rangle$, for some $u' \in U$:
$A(a)(u)$ is defined, $A(a)(u) = u'$ and $S(u') = s'$ and
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle = \emptyset$ then for each $u \in U$:
if $S(u) = s$ then $A(a)(u)$ is undefined
$\iff$ U is a nonempty set,
$S \subseteq U \times \mathcal{S}$,
for each $u \in U$, for some $s \in \mathcal{S}$: $\langle u, s \rangle \in S$,
for each $s_1 \in \mathcal{S}$, for each $s_2 \in \mathcal{S}$:
if $s_1 \neq s_2$ then for each $u \in U$:
it is not the case that $(\langle u, s_1 \rangle \in \mathcal{S}$ and $\langle u, s_2 \rangle \in \mathcal{S})$,
A is a total function from $\mathcal{A}$ to $\text{Pow}(U \times U)$,
for each $a \in \mathcal{A}$, for each $u \in U$, for each $u_1 \in U$, for each $u_2 \in U$:
if $\langle u, u_1 \rangle \in A(a)$ and $\langle u, u_2 \rangle \in A(a)$ then $u_1 = u_2$,
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle \neq \emptyset$ then for each $u \in U$:
if $\langle u, s \rangle \in S$ then for some $s' \in \mathcal{F}\langle s, a \rangle$, for some $u' \in U$:
$\langle u, u' \rangle \in A(a)$ and $\langle u', s' \rangle \in S$ and
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle = \emptyset$ then for each $u \in U$:
if $\langle u, s \rangle \in S$ then it is not the case that for some $u' \in U$:
$\langle u, u' \rangle \in A(a)$
$\iff$ U is a nonempty set,
$\overrightarrow{S}$ is defined,
$\overrightarrow{S}$ is a total function from $\mathcal{S}$ to $\text{Pow}(U)$,
for each $u \in U$, for some $s \in \mathcal{S}$: $u \in \overrightarrow{S}(s)$,
for each $s_1 \in \mathcal{S}$, for each $s_2 \in \mathcal{S}$:
if $s_1 \neq s_2$ then for each $u \in U$:
it is not the case that $(u \in \overrightarrow{S}(s_1)$ and $u \in \overrightarrow{S}(s_2))$,
A is a total function from $\mathcal{A}$ to $\text{Pow}(U \times U)$,
for each $a \in \mathcal{A}$, for each $u \in U$, for each $u_1 \in U$, for each $u_2 \in U$:
if $\langle u, u_1 \rangle \in A(a)$ and $\langle u, u_2 \rangle \in A(a)$ then $u_1 = u_2$,
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle \neq \emptyset$ then for each $u \in U$:
if $u \in \overrightarrow{S}(s)$ then for some $s' \in \mathcal{F}\langle s, a \rangle$, for some $u' \in U$:

$\langle u, u' \rangle \in A(a)$ and $u' \in \overrightarrow{S}(s')$ and
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle = \emptyset$ then for each $u \in U$:
if $u \in \overrightarrow{S}(s)$ then it is not the case that for some $u' \in U$:
$\langle u, u' \rangle \in A(a)$

$\Longleftrightarrow$ $\overrightarrow{S}$ is defined,
$\langle U, \overrightarrow{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$,
for each assignment β in $\langle U, \overrightarrow{S}, A \rangle$, for each $u \in U$, for some $s \in \mathcal{S}$:
$\langle U, \overrightarrow{S}, A \rangle \models_{\beta[x \mapsto u]} s(x)$,
for each $s_1 \in \mathcal{S}$, for each $s_2 \in \mathcal{S}$:
if $s_1 \neq s_2$ then for each assignment β in $\langle U, \overrightarrow{S}, A \rangle$, for each $u \in U$:
$\langle U, \overrightarrow{S}, A \rangle \models_{\beta[x \mapsto u]} \neg(s_1(x) \wedge s_2(x))$,
for each $a \in \mathcal{A}$, for each assignment β in $\langle U, \overrightarrow{S}, A \rangle$, for each $u \in U$, for each $u_1 \in U$, for each $u_2 \in U$:
$\langle U, \overrightarrow{S}, A \rangle \models_{\beta[x \mapsto u][y \mapsto u_1][z \mapsto u_2]} ((a(x, y) \wedge a(x, z)) \rightarrow (y = z))$,
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle \neq \emptyset$ then for each assignment β in $\langle U, \overrightarrow{S}, A \rangle$, for each $u \in U$:
if $\langle U, \overrightarrow{S}, A \rangle \models_{\beta[x \mapsto u]} s(x)$ then for some $s' \in \mathcal{F}\langle s, a \rangle$, for some $u' \in U$:
$\langle U, \overrightarrow{S}, A \rangle \models_{\beta[x \mapsto u][y \mapsto u']} (a(x, y) \wedge s'(y))$ and
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle = \emptyset$ then for each assignment β in $\langle U, \overrightarrow{S}, A \rangle$, for each $u \in U$:
if $\langle U, \overrightarrow{S}, A \rangle \models_{\beta[x \mapsto u]} s(x)$ then it is not the case that for some $u' \in U$:
$\langle U, \overrightarrow{S}, A \rangle \models_{\beta[x \mapsto u][y \mapsto u']} a(x, y)$

$\Longleftrightarrow$ $\overrightarrow{S}$ is defined,
$\langle U, \overrightarrow{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$,
$\langle U, \overrightarrow{S}, A \rangle \models (\forall x \bigvee \{s(x) | s \in \mathcal{S}\})$,
for each $s_1 \in \mathcal{S}$, for each $s_2 \in \mathcal{S}$:
if $s_1 \neq s_2$ then $\langle U, \overrightarrow{S}, A \rangle \models (\forall x \neg(s_1(x) \wedge s_2(x)))$,
for each $a \in \mathcal{A}$:
$\langle U, \overrightarrow{S}, A \rangle \models (\forall x \forall y \forall z((a(x, y) \wedge a(x, z)) \rightarrow (y = z)))$,
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle \neq \emptyset$
then $\langle U, \overrightarrow{S}, A \rangle \models (\forall x(s(x) \rightarrow \bigvee \{(\exists y(a(x, y) \wedge s'(y))) | s' \in \mathcal{F}\langle s, a \rangle\}))$ and
for each $s \in \mathcal{S}$, for each $a \in \mathcal{A}$:
if $\mathcal{F}\langle s, a \rangle = \emptyset$ then $\langle U, \overrightarrow{S}, A \rangle \models (\forall x(s(x) \rightarrow \neg(\exists y\, a(x, y))))$

$\Longleftrightarrow$ $\overrightarrow{S}$ is defined,
$\langle U, \overrightarrow{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$,
$\langle U, \overrightarrow{S}, A \rangle \models \{(\forall x \bigvee \{s(x) | s \in \mathcal{S}\})\}$,
$\langle U, \overrightarrow{S}, A \rangle \models \{(\forall x \neg(s_1(x) \wedge s_2(x))) | s_1 \in \mathcal{S}, s_2 \in \mathcal{S}, s_1 \neq s_2\}$,
$\langle U, \overrightarrow{S}, A \rangle \models \{(\forall x \forall y \forall z((a(x, y) \wedge a(x, z)) \rightarrow (y = z))) | a \in \mathcal{A}\}$,
$\langle U, \overrightarrow{S}, A \rangle \models \{(\forall x(s(x) \rightarrow \bigvee \{(\exists y(a(x, y) \wedge s'(y))) | s' \in \mathcal{F}\langle s, a \rangle\})) | s \in \mathcal{S}, a \in \mathcal{A}, \mathcal{F}\langle s, a \rangle \neq \emptyset\}$ and
$\langle U, \overrightarrow{S}, A \rangle \models \{(\forall x(s(x) \rightarrow \neg(\exists y\, a(x, y)))) | s \in \mathcal{S}, a \in \mathcal{A}, \mathcal{F}\langle s, a \rangle = \emptyset\}$

$\Longleftrightarrow$ $\overrightarrow{S}$ is defined,
$\langle U, \overrightarrow{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$ and
$\langle U, \overrightarrow{S}, A \rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle)$.

□

Descriptions

An SRL signature embodies assertions about SRL interpretations, some of which I termed the s- and a-restrictions. Assertions, like FOL sentences, are either true or false when interpreted. The s- and a-restrictions can therefore be translated into *denotation preserving* FOL sentences. However, this is not true of SRL descriptions. An SRL description denotes a subset of the universe, whereas an FOL sentence denotes a truth value, so no denotation preserving translation of SRL descriptions to FOL sentences is possible. Yet, any translation which assures the right correlation between SRL non-prediction and FOL inference will suffice. Although I may not be able to *preserve* the denotation of an SRL description, I can hope to obtain the appropriate correlation by somehow *capturing* the meaning of the SRL description in an FOL formula. [Kin] explains that

> Underlying a conventional interpretable logic is the intuition that an assertion is a finite and syntactically well-formed string of symbols that is either true or false when interpreted. Underlying SRL is the intuition that a description is a finite and syntactically well-formed string of symbols that is either true or false *of an object* when interpreted. For example, the English sentence 'it is black' is true of a soot particle but false of a snow flake when interpreted. [Kin, chapter 2]

To mimic this intuition in FOL, I select a unique variable x and call an FOL formula f true *of an object* u under an interpretation I iff x is the only free variable in f and some assignment in I that assigns u to x satisfies f in I. Formally,

Definition 37 *Let* $\langle x, y_0, z_0, y_1, z_1, \ldots \rangle$ *be some well-ordering of Var. For each SRL signature* $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$*:*

Desc *is the total function from* $\mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle}$ *to* $\mathcal{F}_{\langle \mathcal{S}, \mathcal{A} \rangle}$ *such that*

$$\begin{aligned}
&\mathsf{Desc}(: a_1 \ldots a_n \sim s) \\
&= (\exists y_0 \ldots \exists y_n ((x = y_0) \wedge a_1(y_0, y_1) \wedge \ldots \wedge a_n(y_{n-1}, y_n) \wedge s(y_n))), \\
&\mathsf{Desc}(: a_1 \ldots a_n \approx : a'_1 \ldots a'_m) \\
&= (\exists y_0 \ldots \exists y_n \exists z_0 \ldots \exists z_m ((x = y_0) \wedge a_1(y_0, y_1) \wedge \ldots \wedge a_n(y_{n-1}, y_n), \\
&\qquad\qquad\qquad \wedge (x = z_0) \wedge a'_1(z_0, z_1) \wedge \ldots \wedge a'_m(z_{m-1}, z_m) \\
&\qquad\qquad\qquad \wedge (y_n = z_m))), \\
&\mathsf{Desc}(\neg d) = \neg \mathsf{Desc}(d), \\
&\mathsf{Desc}((d_1 \wedge d_2)) = (\mathsf{Desc}(d_1) \wedge \mathsf{Desc}(d_2)), \\
&\mathsf{Desc}((d_1 \vee d_2)) = (\mathsf{Desc}(d_1) \vee \mathsf{Desc}(d_2)) \text{ and} \\
&\mathsf{Desc}((d_1 \rightarrow d_2)) = (\mathsf{Desc}(d_1) \rightarrow \mathsf{Desc}(d_2)).
\end{aligned}$$

The translation of an SRL description is an FOL formula that mimics the SRL notion of true *of an object* in FOL: An SRL description from a limited SRL signature is true of an object under an SRL interpretation iff the corresponding FOL interpretation satisfies the translation of the s- and a-restrictions of the signature and the corresponding FOL formula is true of the object under the FOL interpretation.

Proposition 12 *For each limited SRL signature* $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$*, for each triple* $\langle U, S, A \rangle$*, for each* $d \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle}$*, for each* $\beta : Var \rightarrow U$*, for each* $u \in U$*:*

$\langle U, S, A\rangle$ *is an SRL interpretation of* $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ *and*
$u \in [\![d]\!]_{\langle U,S,A\rangle}$
iff $\vec{S}$ *is defined,*
$\langle U, \vec{S}, A\rangle$ *is an FOL interpretation of* $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ *and*
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d)$.

Proof By strong arithmetic induction on the number of $\neg$, $\wedge$, $\vee$ and $\rightarrow$ symbols in d.

Firstly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each triple $\langle U, S, A\rangle$, for each $a_1 \in \mathcal{A}, \ldots$, for each $a_n \in \mathcal{A}$, for each $s \in \mathcal{S}$, for each $\beta : \mathit{Var} \to U$, for each $u \in U$:

$\langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and $u \in [\![: a_1 \ldots a_n \sim s]\!]_{\langle U,S,A\rangle}$
$\iff \langle U, S, A\rangle$ is a nonempty SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$,
$A(a_n)(\ldots A(a_1)(u)\ldots)$ is defined and $S(A(a_n)(\ldots A(a_1)(u)\ldots)) = s$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and by Proposition 11
for some $u_0 \in U, \ldots$, for some $u_n \in U$:
$u = u_0, \langle u_0, u_1\rangle \in A(a_1), \ldots, \langle u_{n-1}, u_n\rangle \in A(a_n)$ and $u_n \in \vec{S}\ (s)$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
for some $u_0 \in U, \ldots$, for some $u_n \in U$:
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u][y_0\mapsto u_0]\ldots[y_n\mapsto u_n]} ((x = y_0) \wedge a_1(y_0, y_1) \wedge \ldots \wedge a_n(y_{n-1}, y_n) \wedge s(y_n))$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} (\exists y_0 \ldots \exists y_n((x = y_0) \wedge a_1(y_0, y_1) \wedge \ldots \wedge a_n(y_{n-1}, y_n) \wedge s(y_n)))$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(: a_1 \ldots a_n \sim s)$. $\ldots 1$

Secondly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each triple $\langle U, S, A\rangle$, for each $a_1 \in \mathcal{A}, \ldots$, for each $a_n \in \mathcal{A}$, for each $a'_1 \in \mathcal{A}, \ldots$, for each $a'_m \in \mathcal{A}$, for each $\beta : \mathit{Var} \to U$, for each $u \in U$:

$\langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and
$u \in [\![: a_1 \ldots a_n \approx : a'_1 \ldots a'_m]\!]_{\langle U,S,A\rangle}$
$\iff \langle U, S, A\rangle$ is a nonempty SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$,
$A(a_n)(\ldots A(a_1)(u)\ldots)$ is defined,
$A(a'_m)(\ldots A(a'_1)(u)\ldots)$ is defined and
$A(a_n)(\ldots A(a_1)(u)\ldots) = A(a'_m)(\ldots A(a'_1)(u)\ldots)$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and by Proposition 11
for some $u_0 \in U, \ldots$, for some $u_n \in U$, for some $u'_0 \in U, \ldots$, for some $u'_m \in U$:
$u = u_0, \langle u_0, u_1\rangle \in A(a_1), \ldots, \langle u_{n-1}, u_n\rangle \in A(a_n)$,
$u = u'_0, \langle u'_0, u'_1\rangle \in A(a'_1), \ldots, \langle u'_{m-1}, u'_m\rangle \in A(a'_m)$ and
$u_n = u'_m$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
for some $u_0 \in U, \ldots$, for some $u_n \in U$, for some $u'_0 \in U, \ldots$, for some $u'_m \in U$:

$$\begin{array}{l}
\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u][y_0\mapsto u_0]\ldots[y_n\mapsto u_n][z_0\mapsto u'_0]\ldots[z_m\mapsto u'_m]} \\
\quad ((x = y_0) \wedge a_1(y_0, y_1) \wedge \ldots \wedge a_n(y_{n-1}, y_n) \\
\quad \wedge (x = z_0) \wedge a'_1(z_0, z_1) \wedge \ldots \wedge a'_m(z_{m-1}, z_m) \\
\quad \wedge (y_n = z_m))
\end{array}$$

$$\begin{array}{ll}
\iff & \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and} \\
& \langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \\
& (\exists y_0 \ldots \exists y_n \exists z_0 \ldots \exists z_m((x = y_0) \wedge a_1(y_0, y_1) \wedge \ldots \wedge a_n(y_{n-1}, y_n) \\
& \qquad\qquad \wedge (x = z_0) \wedge a'_1(z_0, z_1) \wedge \ldots \wedge a'_m(z_{m-1}, z_m) \\
& \qquad\qquad \wedge (y_n = z_m))) \\
\iff & \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and} \\
& \langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(: a_1 \ldots a_n \approx : a'_1 \ldots a'_m). \qquad \ldots 2
\end{array}$$

Thirdly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each triple $\langle U, S, A\rangle$, for each $d \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $\beta : \mathit{Var} \to U$, for each $u \in U$:

$$\begin{array}{ll}
\multicolumn{2}{l}{\langle U, S, A\rangle \text{ is an SRL interpretation of } \langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle \text{ and } u \in [\![\neg d]\!]_{\langle U,S,A\rangle}} \\
\iff & \langle U, S, A\rangle \text{ is a nonempty SRL interpretation of } \langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle \text{ and} \\
& \text{it is not the case that } u \in [\![d]\!]_{\langle U,S,A\rangle} \\
\iff & \langle U, S, A\rangle \text{ is a nonempty SRL interpretation of } \langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle \text{ and} \\
& \text{it is not the case that} \\
& \quad \langle U, S, A\rangle \text{ is an SRL interpretation of } \langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle \text{ and } u \in [\![d]\!]_{\langle U,S,A\rangle} \\
\iff & \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and} \qquad \text{by Proposition 11} \\
& \text{it is not the case that} \\
& \quad \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \quad \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and } \langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d) \\
& \qquad\qquad\qquad\qquad \text{by induction hypothesis} \\
\iff & \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and} \\
& \text{it is not the case that } \langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d) \\
\iff & \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and} \\
& \langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \neg\mathsf{Desc}(d) \\
\iff & \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and} \\
& \langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(\neg d). \qquad \ldots 3
\end{array}$$

Fourthly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each triple $\langle U, S, A\rangle$, for each $d_1 \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $d_2 \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $\beta : \mathit{Var} \to U$, for each $u \in U$:

$$\begin{array}{ll}
\multicolumn{2}{l}{\langle U, S, A\rangle \text{ is an SRL interpretation of } \langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle \text{ and } u \in [\![(d_1 \wedge d_2)]\!]_{\langle U,S,A\rangle}} \\
\iff & \langle U, S, A\rangle \text{ is an SRL interpretation of } \langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle \text{ and} \\
& \quad (u \in [\![d_1]\!]_{\langle U,S,A\rangle} \text{ and } u \in [\![d_2]\!]_{\langle U,S,A\rangle}) \\
\iff & \vec{S} \text{ is defined, } \langle U, \vec{S}, A\rangle \text{ is an FOL interpretation of } \langle \mathcal{S}, \mathcal{A}\rangle, \\
& \langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \text{ and} \qquad \text{by Proposition 11} \\
& \quad (\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d_1) \text{ and } \langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d_2)) \\
& \qquad\qquad\qquad\qquad \text{by induction hypothesis}
\end{array}$$

$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} (\mathsf{Desc}(d_1) \wedge \mathsf{Desc}(d_2))$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}((d_1 \wedge d_2))$. ...4

Fifthly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each triple $\langle U, S, A\rangle$, for each $d_1 \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $d_2 \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $\beta : \mathit{Var} \to U$, for each $u \in U$:
$\langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and $u \in [\![(d_1 \vee d_2)]\!]_{\langle U,S,A\rangle}$
$\iff \langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and
$(u \in [\![d_1]\!]_{\langle U,S,A\rangle}$ or $u \in [\![d_2]\!]_{\langle U,S,A\rangle})$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and by Proposition 11
$(\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d_1)$ or $\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d_2))$
by induction hypothesis
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} (\mathsf{Desc}(d_1) \vee \mathsf{Desc}(d_2))$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}((d_1 \vee d_2))$. ...5

Sixthly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each triple $\langle U, S, A\rangle$, for each $d_1 \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $d_2 \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $\beta : \mathit{Var} \to U$, for each $u \in U$:
$\langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and $u \in [\![(d_1 \to d_2)]\!]_{\langle U,S,A\rangle}$
$\iff \langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and
(if $u \in [\![d_1]\!]_{\langle U,S,A\rangle}$ then $u \in [\![d_2]\!]_{\langle U,S,A\rangle}$)
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and by Proposition 11
(if $\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d_1)$ then $\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d_2)$)
by induction hypothesis
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} (\mathsf{Desc}(d_1) \to \mathsf{Desc}(d_2))$
$\iff \vec{S}$ is defined, $\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}((d_1 \to d_2))$. ...6

Thus, by 1–6 and strong arithmetic induction:
For each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each triple $\langle U, S, A\rangle$, for each $d \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $\beta : \mathit{Var} \to U$, for each $u \in U$:
$\langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and
$u \in [\![d]\!]_{\langle U,S,A\rangle}$
iff $\vec{S}$ is defined,
$\langle U, \vec{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$,
$\langle U, \vec{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle)$ and
$\langle U, \vec{S}, A\rangle \models_{\beta[x\mapsto u]} \mathsf{Desc}(d)$.

□

3.2 Correlation

In order to establish a correlation between SRL non-prediction and FOL inference, I proceed in two steps. Firstly, in Proposition 13, I establish a correlation between SRL prediction and FOL satisfiability. Then, in Proposition 14, I use standard FOL results to establish from Proposition 13 a correlation between SRL non-prediction and FOL inference.

Lemma 1 *For each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$, for each triple $\langle U, S, A \rangle$, for each $\theta \subseteq \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle}$, for each $d \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle}$:*
$\langle U, S, A \rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$,
$\langle U, S, A \rangle$ SRL models θ under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$ and
$\langle U, S, A \rangle$ SRL satisfies d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$
iff $\vec{S}$ is defined,
$\langle U, \vec{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$ and
$\langle U, \vec{S}, A \rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$.

Proof For each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$, for each triple $\langle U, S, A \rangle$, for each $\theta \subseteq \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle}$, for each $d \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle}$:
$\langle U, S, A \rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$,
$\langle U, S, A \rangle$ SRL models θ under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$ and
$\langle U, S, A \rangle$ SRL satisfies d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$
$\iff$ $\langle U, S, A \rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$,
$[\![\theta]\!]_{\langle U,S,A \rangle} = U$ and $[\![d]\!]_{\langle U,S,A \rangle} \neq \emptyset$
$\iff$ $\langle U, S, A \rangle$ is a nonempty SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle$,
for each $d' \in \theta$, for each $u' \in U$:
$u' \in [\![d']\!]_{\langle U,S,A \rangle}$ and
for some $u \in U$:
$u \in [\![d]\!]_{\langle U,S,A \rangle}$
$\iff$ $\vec{S}$ is defined,
$\langle U, \vec{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$,
$\langle U, \vec{S}, A \rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle)$, by Proposition 11
for each $d' \in \theta$, for each assignment β in $\langle U, \vec{S}, A \rangle$, for each $u' \in U$:
$\langle U, \vec{S}, A \rangle \models_{\beta[x \mapsto u']} \mathsf{Desc}(d')$ and
for each assignment β in $\langle U, \vec{S}, A \rangle$, for some $u \in U$:
$\langle U, \vec{S}, A \rangle \models_{\beta[x \mapsto u]} \mathsf{Desc}(d)$ by Proposition 12
$\iff$ $\vec{S}$ is defined,
$\langle U, \vec{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$,
$\langle U, \vec{S}, A \rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle)$,
for each $d' \in \theta$, for each assignment β in $\langle U, \vec{S}, A \rangle$:
$\langle U, \vec{S}, A \rangle \models_{\beta} (\forall x \mathsf{Desc}(d'))$ and
for each assignment β in $\langle U, \vec{S}, A \rangle$:
$\langle U, \vec{S}, A \rangle \models_{\beta} (\exists x \mathsf{Desc}(d))$
$\iff$ $\vec{S}$ is defined,
$\langle U, \vec{S}, A \rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A} \rangle$ and
$\langle U, \vec{S}, A \rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F} \rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$.

□

Definition 38 *For each Y, for each Z, for each total function F from Z to* $\mathrm{Pow}(Y)$*:*
$\overleftarrow{F} \subseteq Y \times Z$ *such that*
for each $y \in Y$, for each $z \in Z$: $\langle y, z\rangle \in \overleftarrow{F}$ iff $y \in F(z)$.

Lemma 2 *For each Y, for each Z, for each total function F from Z to* $\mathrm{Pow}(Y)$*:*
$\overset{\rightleftarrows}{F} = F$.

Proof For each Y, for each Z, for each total function F from Z to $\mathrm{Pow}(Y)$, for each $z \in Z$:
$\overset{\rightleftarrows}{F}(z) = \{y \in Y | \langle y, z\rangle \in \overleftarrow{F}\} = \{y \in Y | y \in F(z)\} = F(z)$.
Thus,
for each Y, for each Z, for each total function F from Z to $\mathrm{Pow}(Y)$: $\overset{\rightleftarrows}{F} = F$.
□

Proposition 13 *For each limited SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:*
θ SRL predicts d under Σ
iff $\mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$ *is FOL satisfiable.*

Proof Firstly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each $\theta \subseteq \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $d \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$:
θ SRL predicts d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$
$\Longrightarrow$ for some SRL interpretation I of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$:
I SRL models θ under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and I SRL satisfies d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$
$\Longrightarrow$ for some triple $\langle U, S, A\rangle$:
$\langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$,
$\langle U, S, A\rangle$ SRL models θ under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and $\langle U, S, A\rangle$ SRL satisfies d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$
$\Longrightarrow$ for some triple $\langle U, S, A\rangle$:
$\overrightarrow{S}$ is defined,
$\langle U, \overrightarrow{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$ and
$\langle U, \overrightarrow{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$
by Lemma 1
$\Longrightarrow$ for some FOL interpretation I of $\langle \mathcal{S}, \mathcal{A}\rangle$:
$I \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$
$\Longrightarrow$ $\mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$ is FOL satisfiable.
...1

Secondly, for each limited SRL signature $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$, for each $\theta \subseteq \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$, for each $d \in \mathcal{D}_{\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle}$:
$\mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$ is FOL satisfiable
$\Longrightarrow$ for some FOL interpretation I of $\langle \mathcal{S}, \mathcal{A}\rangle$:
$I \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$
$\Longrightarrow$ for some triple $\langle U, S', A\rangle$:
$\langle U, S', A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$ and
$\langle U, S', A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$
$\Longrightarrow$ for some triple $\langle U, S', A\rangle$:

$\overleftarrow{S'}$ is defined, $\overset{\rightleftarrows}{S'}$ is defined,
$\langle U, \overset{\rightleftarrows}{S'}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$ and
$\langle U, \overset{\rightleftarrows}{S'}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d'))|d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$
by Lemma 2

$\Longrightarrow$ for some triple $\langle U, S, A\rangle$:
$\overrightarrow{S}$ is defined,
$\langle U, \overrightarrow{S}, A\rangle$ is an FOL interpretation of $\langle \mathcal{S}, \mathcal{A}\rangle$ and
$\langle U, \overrightarrow{S}, A\rangle \models \mathsf{Restrict}(\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle) \cup \{(\forall x \mathsf{Desc}(d'))|d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$
substitute S for $\overleftarrow{S'}$

$\Longrightarrow$ for some triple $\langle U, S, A\rangle$:
$\langle U, S, A\rangle$ is an SRL interpretation of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$,
$\langle U, S, A\rangle$ SRL models θ under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and $\langle U, S, A\rangle$ SRL satisfies d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ by Lemma 1

$\Longrightarrow$ for some SRL interpretation I of $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$:
I SRL models θ under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$ and I SRL satisfies d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$

$\Longrightarrow$ θ SRL predicts d under $\langle \mathcal{S}, \mathcal{A}, \mathcal{F}\rangle$. ...2

Thus, by 1 and 2:
For each limited SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:
θ SRL predicts d under Σ
iff $\mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d'))|d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$ is FOL satisfiable.

□

Notice in passing that two corollaries of Proposition 13 establishes a correlation between SRL satisfiability and FOL satisfiability as well as between SRL modellability and FOL satisfiability. An SRL description is SRL satisfiable iff an FOL theory corresponding to the description is FOL satisfiable.

Corollary 1 *For each limited SRL signature Σ, for each $d \in \mathcal{D}_\Sigma$:*
d is SRL satisfiable under Σ iff $\mathsf{Restrict}(\Sigma) \cup \{(\exists x \mathsf{Desc}(d))\}$ *is FOL satisfiable.*

Proof For each limited SRL signature Σ, for each $d \in \mathcal{D}_\Sigma$:
d is SRL satisfiable under Σ
$\Longleftrightarrow$ $\emptyset$ SRL predicts d under Σ by Proposition 8
$\Longleftrightarrow$ $\mathsf{Restrict}(\Sigma) \cup \{(\exists x \mathsf{Desc}(d))\}$ is FOL satisfiable. by Proposition 13

□

An SRL theory is SRL modellable iff an FOL theory corresponding to the SRL theory is FOL satisfiable.

Corollary 2 *For each limited SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$:*
θ is SRL modellable under Σ iff $\mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d))|d \in \theta\}$ *is FOL satisfiable.*

Proof For each limited SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$:
θ is SRL modellable under Σ
$\Longleftrightarrow$ θ SRL predicts $:\approx:$ under Σ by Proposition 9
$\Longleftrightarrow$ $\mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d))|d \in \theta\} \cup \{(\exists x \exists y_0 \exists z_0((x = y_0) \wedge (x = z_0) \wedge (y_0 = z_0)))\}$ is FOL satisfiable by Proposition 13
$\Longleftrightarrow$ $\mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d))|d \in \theta\}$ is FOL satisfiable.

□

Finally, Proposition 13 and standard FOL results also establish a correlation between SRL non-prediction and FOL inference. An SRL theory does not SRL predict an SRL description iff an FOL theory corresponding to the SRL theory infers an FOL sentence corresponding to the SRL description.

Proposition 14 *For each limited SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:*

θ does not SRL predict d under Σ

iff $\mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \vdash \neg(\exists x \mathsf{Desc}(d))$.

Proof For each limited SRL signature Σ, for each $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:

θ does not SRL predict d under Σ

$\Longleftrightarrow \mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \cup \{(\exists x \mathsf{Desc}(d))\}$ is not FOL satisfiable

by Proposition 13

$\Longleftrightarrow \mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \models \neg(\exists x \mathsf{Desc}(d))$ by Proposition 3

$\Longleftrightarrow \mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \vdash \neg(\exists x \mathsf{Desc}(d))$. by Proposition 4

□

Chapter 4

Calculus

In this chapter, I argue that a sound and complete calculus for SRL prediction is logically inconceivable. More precisely, for *some* SRL signature, there is no sound and complete calculus such that for each SRL theory and each SRL description from the signature, the theory predicts the description iff the calculus furnishes a proof that the theory predicts the description.

The chapter comprises two sections. In the first section, I argue for the thesis that, under certain conditions, any sound and complete calculus for a consequence relation generates a demonstrable proof set for the relation. In the second section, I prove that there is no demonstrable proof set for SRL prediction and thus arrive at the thesis that there is no sound and complete calculus for SRL prediction.

4.1 Theses

It is well-established that the need for a sound and complete calculus stems from the need to precisely define the notion of a *mathematical proof*.

Thesis 3 *A sound and complete calculus for a consequence relation defines precisely the notion of mathematical proof, i.e. what constitutes a proof of a conclusion following from some premises.*

I call a set *finitary* iff each member of the set (but possibly not the set itself) is finite. As already mentioned in section 1.1, a proof must convince others, i.e. the proofhood of a candidate proof of a conclusion following from some premises must be scrutable by an effective method, at least for a countable and finitary language[1], finite sets of premises from the language and a given countable and finitary set of proofs by which proofhood is defined.

> A basic concept of logic is that of a *proof*. Now a proof, viewed abstractly, is a series of statements that "establishes" without doubt the truth of its conclusion, given the truth of its assumptions. But to establish convincingly the truth of the conclusion, the proof must be verifiable by others. There must be some procedure by which an outsider can verify the correctness of the proof, without having to supply brilliant insight. That is, it must be

[1]Thus far, a language is finitary *by definition*, but this definition is sometimes generalized to include sets of infinite 'strings'.

possible to verify the correctness of proofs by an effective procedure. The set of proofs must be recursive. [End77, page 546]

Thesis 4 *In a countable and finitary language, membership in a given countable and finitary set of proofs of any candidate proof of a conclusion following from a finite set of premises must be scrutable by an effective method, i.e. the set of proofs from finite sets of premises must be Turing decidable.*

The following definition of a *demonstrable proof set* for a binary relation between theories and assertions in a countable language incorporates these ideas: A Turing decidable set of finite-theory/assertion/argument triples such that the proof set contains a triple iff the theory and assertion stand in the relation. Since the proof set contains triples for precisely the finitary part of the relation, each theory/assertion/argument triple constitutes a *proof* that the theory and assertion stand in the relation.

Definition 39 *For each alphabet $\mathcal{A}$, for each $\mathcal{D} \subseteq \mathcal{A}^*$, for each $r \subseteq \mathrm{Pow}(\mathcal{D}) \times \mathcal{D}$, for each P:*

P is a **demonstrable proof set** *for r*
iff $P \subseteq \mathrm{FinPow}(\mathcal{D}) \times \mathcal{D} \times \mathcal{A}^*$,
for each $\theta \in \mathrm{FinPow}(\mathcal{D})$, for each $d \in \mathcal{D}$:
$\langle \theta, d \rangle \in r$ iff for some $\Delta \in \mathcal{A}^*$: $\langle \theta, d, \Delta \rangle \in P$ and
P is Turing decidable.

I call a sound and complete calculus *finitary* iff the calculus defines a finitary set of proofs. From Thesis 3 and Thesis 4, I arrive at the following thesis.

Thesis 5 *In a countable and finitary language, a finitary, sound and complete calculus for a consequence relation generates a demonstrable proof set for the relation.*

Notice first that the most prototypical examples of sound and complete calculuses — a sound and complete calculus for propositional logic entailment or FOL entailment between theories and formulae in a countable language — generate demonstrable proof sets for propositional logic entailment or FOL entailment, respectively. Notice further that Thesis 5 makes no commitment to whether the following calculuses should or should not generate demonstrable proof sets: Calculuses for propositional logic entailment or FOL entailment in uncountable languages; calculuses for entailment of infinitary logics; or calculuses generating infinite proofs.

4.2 Prediction

In order to prove that there is no demonstrable proof set for SRL prediction under some SRL signature, I proceed in three steps. Firstly, in Proposition 15, I prove that if a demonstrable proof set for a binary relation in a finitary and countable language exists, then the finitary part of the relation is Turing acceptable. Secondly, in Proposition 16 and Proposition 17, I show that SRL prediction under some finite SRL signature is not Turing decidable and SRL non-prediction under each finite SRL signature is Turing acceptable. From this it immediately follows, in Proposition 18, that SRL prediction under some finite SRL signature is not Turing acceptable. Finally, Theorem 1 uses these two results to establish that there is no demonstrable proof set for SRL prediction under some finite SRL signature.

If there exists a demonstrable proof set for a binary relation in a finitary and countable language then the finitary part of the relation is Turing acceptable.

Proposition 15 *For each alphabet $\mathcal{A}$, for each $\mathcal{D} \subseteq \mathcal{A}^*$, for each $r \subseteq \mathrm{Pow}(\mathcal{D}) \times \mathcal{D}$, for each P:*

if P is a demonstrable proof set for r

then $\{\langle \theta, d\rangle \in \mathrm{FinPow}(\mathcal{D}) \times \mathcal{D} | \langle \theta, d\rangle \in r\}$ is Turing acceptable.

Proof For each alphabet $\mathcal{A}$, for each $\mathcal{D} \subseteq \mathcal{A}^*$, for each $r \subseteq \mathrm{Pow}(\mathcal{D}) \times \mathcal{D}$, for each P:

P is a demonstrable proof set for r
$\Longrightarrow P$ is Turing decidable
$\Longrightarrow P$ is Turing acceptable — by Proposition 1
$\Longrightarrow P$ is Turing enumerable — by Proposition 2
$\Longrightarrow$ for some Turing machine $\mathfrak{M}_P$:
$\mathfrak{M}_P$ enumerates P.

For each alphabet $\mathcal{A}$, for each $\mathcal{D} \subseteq \mathcal{A}^*$, for each $r \subseteq \mathrm{Pow}(\mathcal{D}) \times \mathcal{D}$, for each demonstrable proof set P for r, let $\mathfrak{M}$ be a Turing machine such that for each input string $\langle \theta, d\rangle \in \mathrm{FinPow}(\mathcal{D}) \times \mathcal{D}$:

1. $\mathfrak{M}$ runs $\mathfrak{M}_P$ to enumerate some $\langle \theta', d', \Delta'\rangle \in \mathrm{FinPow}(\mathcal{D}) \times \mathcal{D} \times \mathcal{A}^*$.
2. If $\theta = \theta'$ and $d = d'$ then $\mathfrak{M}$ halts, otherwise $\mathfrak{M}$ goes to step 1.

$\mathfrak{M}$ halts on input $\langle \theta, d\rangle \in \mathrm{FinPow}(\mathcal{D}) \times \mathcal{D}$ iff for some $\Delta \in \mathcal{A}^*$: $\langle \theta, d, \Delta\rangle \in P$.
Thus,
for each alphabet $\mathcal{A}$, for each $\mathcal{D} \subseteq \mathcal{A}^*$, for each $r \subseteq \mathrm{Pow}(\mathcal{D}) \times \mathcal{D}$, for each P:
P is a demonstrable proof set for r
$\Longrightarrow$ for some Turing machine $\mathfrak{M}$, for each finite $\theta \subseteq \mathcal{D}$, for each $d \in \mathcal{D}$:
$\mathfrak{M}$ halts on input $\langle \theta, d\rangle$ iff $\langle \theta, d\rangle \in r$.
$\Longrightarrow$ for some Turing machine $\mathfrak{M}$:
$\mathfrak{M}$ accepts $\{\langle \theta, d\rangle \in \mathrm{FinPow}(\mathcal{D}) \times \mathcal{D} | \langle \theta, d\rangle \in r\}$.
$\Longrightarrow \{\langle \theta, d\rangle \in \mathrm{FinPow}(\mathcal{D}) \times \mathcal{D} | \langle \theta, d\rangle \in r\}$ is Turing acceptable.

□

SRL prediction is not Turing decidable, not even under finite SRL signatures.

Proposition 16 *For some finite SRL signature Σ:*

$\{\langle \theta, d\rangle \in \mathrm{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$ is not Turing decidable.

Proof For some finite SRL signature Σ:
$\{d \in \mathcal{D}_\Sigma | d$ is SRL modellable under $\Sigma\}$ is Π^0_1-hard. — by Proposition 7
Thus,
for some finite SRL signature Σ:
$\{d \in \mathcal{D}_\Sigma | d$ is SRL modellable under $\Sigma\}$ is not Turing decidable.
by a standard result

Thus,
for some finite SRL signature Σ:
$\{d \in \mathcal{D}_\Sigma | \{d\}$ is SRL modellable under $\Sigma\}$ is not Turing decidable.
Thus,
for some finite SRL signature Σ:
$\{d \in \mathcal{D}_\Sigma | \{d\}$ SRL predicts :$\approx$: under $\Sigma\}$ is not Turing decidable.
by Proposition 9
Thus,
for some finite SRL signature Σ:
$\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$ is not Turing decidable.
$\square$

SRL non-prediction under finite SRL signatures is Turing acceptable .

Proposition 17 *For each finite SRL signature Σ:*
$\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ *does not SRL predict d under* $\Sigma\}$ *is Turing acceptable.*

Proof Firstly, for each finite FOL signature $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{R}, \text{Ar}\rangle$, let
$\mathfrak{C}$ be the calculus in section 2.2 of [Bar77],
$\mathcal{A} = \mathcal{C} \cup \mathcal{F} \cup \mathcal{R} \cup \{\wedge, \vee, \rightarrow, \neg, \forall, \exists, (,), x, 0, \ldots, 9\} \cup$ the finite set of annotation symbols from $\mathfrak{C}$[2] and
$P = \{\langle\theta, f, \Delta\rangle \in \text{FinPow}(\mathcal{F}_\Sigma) \times \mathcal{F}_\Sigma \times \mathcal{A}^* |\ \Delta$ is an inference of f from θ by $\mathfrak{C}\}$.
Clearly, for each finite FOL signature Σ:
P is a demonstrable proof set for $\vdash$.
Thus,
for each finite FOL signature Σ:
$\{\langle\theta, f\rangle \in \text{FinPow}(\mathcal{F}_\Sigma) \times \mathcal{F}_\Sigma | \theta \vdash f\}$ is Turing acceptable.[3] by Proposition 15
Thus,
for each finite FOL signature Σ, for some Turing machine $\mathfrak{M}_P$:
$\mathfrak{M}_P$ accepts $\{\langle\theta, f\rangle \in \text{FinPow}(\mathcal{F}_\Sigma) \times \mathcal{F}_\Sigma | \theta \vdash f\}$.
Thus,
for each finite FOL signature Σ, for some Turing machine $\mathfrak{M}_P$, for each finite sentential $\theta \subseteq \mathcal{F}_\Sigma$, for each sentence $f \in \mathcal{F}_\Sigma$:
$\mathfrak{M}_P$ halts on input $\langle\theta, f\rangle$ iff $\theta \vdash f$.

Secondly, it is also clear that for each finite SRL signature Σ, there exists a Turing machine $\mathfrak{M}$ such that for each input string $\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma$:
$\mathfrak{M}$ first outputs $\langle \mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\}, \neg(\exists x \mathsf{Desc}(d))\rangle$ and then runs $\mathfrak{M}_P$ on that output.
Thus,
for each finite SRL signature Σ, for some Turing machine $\mathfrak{M}$, for each finite $\theta \subseteq \mathcal{D}_\Sigma$, for each $d \in \mathcal{D}_\Sigma$:
$\mathfrak{M}$ halts on input $\langle\theta, d\rangle$
$\iff \mathsf{Restrict}(\Sigma) \cup \{(\forall x \mathsf{Desc}(d')) | d' \in \theta\} \vdash \neg(\exists x \mathsf{Desc}(d))$.
$\iff \theta$ does not SRL predict d under Σ. by Proposition 14
Thus,

[2] Each variable in the countably infinite set of FOL variables *Var* is encoded in the obvious manner as a string from the symbols $x, 0, \ldots, 9$.
[3] This is also a standard result.

for each finite SRL signature Σ, for some Turing machine $\mathfrak{M}$:
$\mathfrak{M}$ accepts $\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ does not SRL predict d under $\Sigma\}$

Thus,
for each finite SRL signature Σ:
$\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ does not SRL predict d under $\Sigma\}$ is Turing acceptable.

□

SRL prediction is not Turing acceptable, not even under finite SRL signatures.

Proposition 18 *For some finite SRL signature Σ:*
$\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ *SRL predicts d under $\Sigma\}$ is not Turing acceptable.*

Proof For some finite SRL signature Σ:
$\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$ is not Turing decidable.
by Proposition 16

Thus,
for some finite SRL signature Σ:
$\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$ is not Turing acceptable.
by Proposition 1 and Proposition 17

□

There is no demonstrable proof set for SRL prediction under finite SRL signatures.

Theorem 1 *For some finite SRL signature Σ, for each P:*
P is not a demonstrable proof set for $\{\langle\theta, d\rangle \in \text{Pow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$.

Proof By contradiction.
For each finite SRL signature Σ, for some P:
P is a demonstrable proof set for $\{\langle\theta, d\rangle \in \text{Pow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$.
$\Longrightarrow$ for each finite SRL signature Σ:
$\{\langle\theta, d\rangle \in \text{FinPow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$ is Turing acceptable
by Proposition 15
$\Longrightarrow$ contradiction. by Proposition 18

Thus,
for some finite SRL signature Σ, for each P:
P is not a demonstrable proof set for $\{\langle\theta, d\rangle \in \text{Pow}(\mathcal{D}_\Sigma) \times \mathcal{D}_\Sigma | \theta$ SRL predicts d under $\Sigma\}$.

□

From Thesis 5 and Theorem 1, I arrive at the following thesis.

Thesis 6 *There is no finitary, sound and complete calculus for SRL prediction under finite SRL signatures.*

Chapter 5

Coda

In this chapter, I investigate the consequences that the lack of a decidability method for SRL prediction, the lack of a finitary, sound and complete calculus for SRL prediction and the existence of a finitary, sound and complete calculus for SRL non-prediction, sketched below, has for the falsification of SRL theories with respect to natural languages.

[PS94] views an HPSG theory to be true of a natural language iff the theory precisely specifies the linguistic objects in the language. The theory may then be falsified with respect to the natural language either (i) by demonstrating that some prediction of the theory describes some object not in the language or (ii) by demonstrating that some object in the language is not described by any prediction of the theory. I call (i) a falsification of the theory due to *overlicensing* and (ii) a falsification of the theory due to *underlicensing*. Note that a falsification of a theory with respect to a natural language due to overlicensing requires a *negative data base*, a collection of objects not in the language, whereas a falsification due to underlicensing requires a *positive database*, a collection of objects in the language.

A decidability method, a finitary, sound and complete calculus for HPSG prediction and a finitary, sound and complete calculus for HPSG non-prediction would then provide means to facilitate the falsification of an HPSG theory with respect to a natural language. For example, in order to employ a decidability method for HPSG prediction to falsify an HPSG theory due to overlicensing, the method would provide a demonstration that some description of some object not in the language is a prediction of the theory. In the case of falsifying an HPSG theory due to underlicensing, the method would provide some demonstration that some description of some object in the language is not a prediction of the theory. Much alike, in order to employ a finitary, sound and complete calculus for prediction to falsify an HPSG theory due to overlicensing, we would provide a demonstration from the calculus that some description of some object not in the language is a prediction of the theory. However, such a calculus would be of no use to falsify an HPSG theory due to underlicensing, since it would lack a demonstration that some description of some object in the language is not a prediction of the theory. The converse would be true for a finitary, sound and complete calculus for HPSG non-prediction: It would enable to falsify an HPSG theory due to underlicensing, but not due to overlicensing.

The formalism of [Kin] cannot have a decidability method or a finitary, sound and complete calculus for SRL prediction, but does possess for each finite SRL signature a finitary, sound and complete calculus for SRL non-prediction — the Restrict and Desc translation functions rendered in a suitable Hilbert-Ackermann style format conjoined with the set of inference rules of the calculus of [Bar77]. These three facts have the following consequences.

Firstly, the impossibility of a decidability method (a finitary, sound and complete calculus) for SRL prediction under some finite SRL signature excludes for arbitrary, even arbitrary finite, SRL signatures, the existence of one particular, algorithmic way (one particular proof language) to falsify arbitrary SRL theories from the signature with respect to arbitrary natural languages, both due to overlicensing and underlicensing (due to overlicensing). Yet, it does not necessarily exclude specific ways (specific demonstrations) to falsify specific SRL theories from specific SRL signatures with respect to specific natural languages due to either overlicensing or underlicensing (due to overlicensing). Moreover, there might well be decidability methods or finitary, sound and complete calculuses for SRL prediction under SRL signatures that meet certain conditions, or for SRL prediction under arbitrary SRL signatures of SRL theories that meet certain conditions. The search for such signatures or theories, their characterization and the specification of appropriate decidability methods or calculuses is a topic of current research.[1]

Secondly, if there is no reason to prefer a falsification of a theory due to overlicensing over a falsification due to underlicensing, then there is no reason to prefer a finitary, sound and complete calculus for SRL prediction over a finitary, sound and complete calculus for SRL non-prediction. Since the type of database determines which type of falsification is applicable, hence which type of calculus is applicable, a positive database enables only the application of a sound and complete calculus for SRL non-prediction, but not one for SRL prediction. By asserting that

> ... many linguists believe strongly in the significance of negative data in the form of native speaker judgments that such-and-such a string cannot be a grammatical sentence of French or Korean or Chinese. But in celestial mechanics there is no negative data: astronomers never get to observe planetary motions accompanied by such glosses as "this can't happen" or "this trajectory is highly marked". [Pol93, page 303],

[Pol93] expresses the common view that the science of natural language possesses a means to obtain negative data as well as positive data, hence in principle allows the application of either a finitary, sound and complete calculus for SRL prediction or one for SRL non-prediction. Yet, this is not true for another commonly used means to obtain linguistic data. Corpora analysis, for example, provides only positive data, hence only enables the application of a finitary, sound and complete calculus for SRL non-prediction for the falsification of an SRL theory with respect to a natural language.

In sum,

1. SRL extended with the SRL prediction relation provides a formalism for HPSG,

but the formalism does not possess two desirable properties, since

2. there is no decidability method for SRL prediction,

[1] For example, [Göt] defines an SRL theory to have the *finite model property* iff for each prediction of the theory, for some SRL interpretation with a finite universe, the interpretation SRL models the theory and SRL satisfies the prediction; and then proves that SRL prediction under finite SRL signatures is Turing decidable for SRL theories with the finite model property.

3. there is no finitary, sound and complete calculus for SRL prediction;

and thus fails to provide a means to facilitate the falsification of an SRL theory with respect to a natural language due to overlicensing. Yet, the formalism *does* possess one desirable property, since

4. there is a finitary, sound and complete calculus for SRL non-prediction,

hence a means to facilitate the falsification of an SRL theory with respect to a natural language due to underlicensing.

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