From Unification to Constraint:
An Evolving Formalism for
Head-driven Phrase Structure Grammar

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Contents

1 Mathematical Preliminaries

1.1 Preorders

1 Definition. *P is a* **preorder** *iff*

P is an ordered pair $\langle S, \circ \rangle$,
S is a set,
$\circ$ is a binary relation on S,
for each $x \in S$, $x \circ x$, and
for each $x \in S$, for each $y \in S$, for each $z \in S$,
if $x \circ y$ and $y \circ z$ then $x \circ z$.

2 Example. *If $S = \{a, b, c, d\}$ and $x \circ y$ iff there is a sequence (possibly empty) of arrows from x to y in*

then $\langle S, \circ \rangle$ is a preorder.

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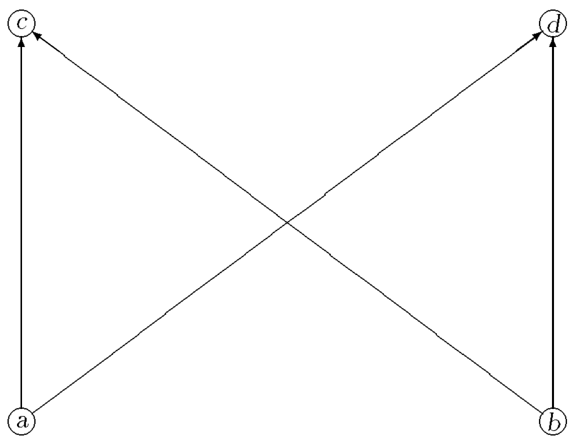
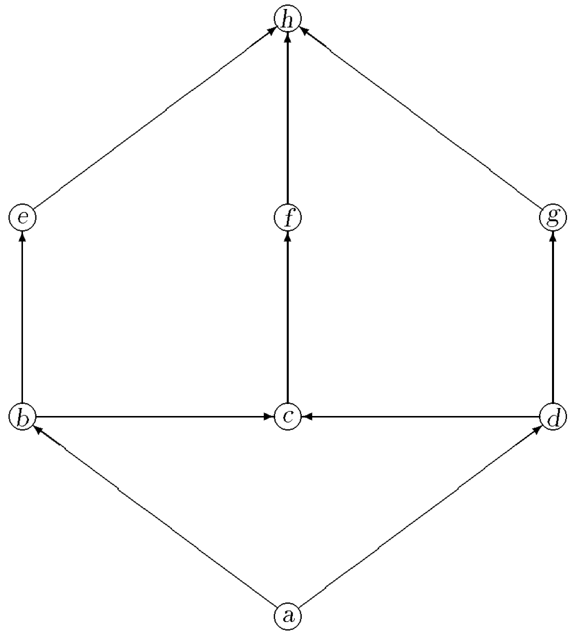


then $\langle S, \circ \rangle$ is a preorder.

4 Example. *If $S = \{a, b, c, d, e, f, g, h\}$ and $x \circ y$ iff there is a sequence (possibly empty) of arrows from x to y in*



then $\langle S, \circ \rangle$ *is a preorder.*

5 Example. *If* $\mathbb{N}$ *is the set of natural numbers and* $x|y$ *iff for some* $z \in \mathbb{N}$, $xz = y$ *then* $\langle \mathbb{N}, | \rangle$ *is a preorder.*

6 Example. *For each set* S, $\langle \mathrm{Pow}(S), \subseteq \rangle$ *is a preorder,*

7 Definition. *For each preorder* $\langle S, \circ \rangle$,

D is a **downward-closed subset** *of* $\langle S, \circ \rangle$ *iff*

$D \subseteq S$, *and*

for each $x \in S$, *for each* $y \in D$, *if* $x \circ y$ *then* $x \in D$.

If P is a preorder then I write $\mathrm{Down}(P)$ for the set of downward-closed subsets of P.

8 Example. *For each preorder* $\langle S, \circ \rangle$, $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$ *is a preorder.*

9 Definition. *For each preorder* $\langle S, \circ \rangle$,

x *is a* **top** *in* $\langle S, \circ \rangle$ *iff*

$x \in S$, *and*

for each $y \in S$, $y \circ x$, *and*

x *is a* **bottom** *in* $\langle S, \circ \rangle$ *iff*

$x \in S$, *and*

for each $y \in S$, $x \circ y$.

10 Examples. *In the preorder of example 2,*

c and d are the only tops, and

there are no bottoms.

In the preorder of example 3,

there are no tops or bottoms.

In the preorder of example 4,

h is the only top, and

a is the only bottom.

In the preorder of example 5,

0 is the only top, and

1 is the only bottom.

For each set S,

S *is the only top in preorder* $\langle \mathrm{Pow}(S), \subseteq \rangle$, *and*

$\emptyset$ *is the only bottom in preorder* $\langle \mathrm{Pow}(S), \subseteq \rangle$.

For each preorder $\langle S, \circ \rangle$,

S *is the only top in preorder* $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$, *and*

$\emptyset$ *is the only bottom in preorder* $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$.

11 Definition. *For each preorder* P,

P *is* **bounded** *iff there is a top and a bottom in* P.

12 Examples. *The preorders in examples 4, 5, 6 and 8 are bounded, but the preorders in examples 2 and 3 are unbounded.*

13 Definition. *For each preorder* $\langle S, \circ \rangle$,

x *is an* **upper bound** *of* Z *in* $\langle S, \circ \rangle$ *iff*

$x \in S$,

$Z \subseteq S$, and

for each $y \in Z$, $y \circ x$, and

x *is a* **lower bound** *of* Z *in* $\langle S, \circ \rangle$ *iff*

$x \in S$,

$Z \subseteq S$, and

for each $y \in Z$, $x \circ y$.

14 Definition. *For each preorder* $\langle S, \circ \rangle$,

x *is a* **least upper bound** *of* Z *in* $\langle S, \circ \rangle$ *iff*

x *is an upper bound of* Z *in* $\langle S, \circ \rangle$, *and*

for each $y \in S$, *if* y *is an upper bound of* Z *in* $\langle S, \circ \rangle$ *then* $x \circ y$, *and*

x *is a* **greatest lower bound** *of* Z *in* $\langle S, \circ \rangle$ *iff*

x *is a lower bound of* Z *in* $\langle S, \circ \rangle$, *and*

for each $y \in S$, *if* y *is a lower bound of* Z *in* $\langle S, \circ \rangle$ *then* $y \circ x$.

15 Examples. *In the preorder of example 2,*

the upper bounds of $\{c, d\}$ *are* c *and* d,

the least upper bounds of $\{c, d\}$ *are* c *and* d,

the lower bounds of $\{c, d\}$ *are* a, b, c *and* d, *and*

the greatest lower bounds of $\{c, d\}$ *are* c *and* d.

In the preorder of example 3,

$\{c, d\}$ *has no upper bounds (and hence no least upper bounds),*

the lower bounds of $\{c, d\}$ *are* a *and* b, *and*

$\{c, d\}$ *has no greatest lower bounds.*

In the preorder of example 4,

the upper bounds of $\{b, d\}$ *are* c, f *and* h,

the only least upper bound of $\{b, d\}$ *is* c, *and*

the only lower bound (and hence only greatest lower bound) of $\{b, d\}$ *is* a.

In the preorder of example 5,

for each $x \in \mathbb{N} \setminus \{0\}$, *for each* $y \in \mathbb{N} \setminus \{0\}$,

the only least upper bound of $\{x, y\}$ *is the lowest common multiple of* x *and* y, *and*

the only greatest lower bound of $\{x, y\}$ *is the highest common factor of* x *and* y, *and*

for each $x \in \mathbb{N}$,

the only least upper bound of $\{x, 0\}$ *is* 0, *and*

the only greatest lower bound of $\{x, 0\}$ *is* x.

For each set S, *for each* $Z \subseteq \mathrm{Pow}(S)$,

$\bigcup Z$ *is the only least upper bound of* Z *in* $\langle \mathrm{Pow}(S), \subseteq \rangle$, *and*

$\bigcap Z$ *is the only greatest lower bound of* Z *in* $\langle \mathrm{Pow}(S), \subseteq \rangle$.[1]

For each preorder $\langle S, \circ \rangle$, *for each* $Z \subseteq \mathrm{Down}(\langle S, \circ \rangle)$,

$\bigcup Z$ *is the only least upper bound of* Z *in* $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$, *and*

$\bigcap Z$ *is the only greatest lower bound of* Z *in* $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$.

16 Definition. *For each preorder* $\langle S, \circ \rangle$,

$\langle S, \circ \rangle$ *is* **upward Noetherian** *iff*

for each $Z \subseteq S$,

if for each $x \in Z$, *for some* $y \in Z$,

$x \circ y$ and $y \not\circ x$

then $Z = \emptyset$.

17 Proposition. *For each preorder* $\langle S, \circ \rangle$,

if S *is finite then* $\langle S, \circ \rangle$ *is upward Noetherian.*

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[1] Let $\bigcap \emptyset = S$.

18 Examples. *The preorders in examples 2, 3 and 4 are all upward Noetherian. However, the preorder of example 5 is not upward Noetherian, since if*

$Z = \{2^i \mid i \text{ is a natural number}\}$

then

$Z \subseteq \mathbb{N}$, *and*

for each $x \in Z$, *for some* $y \in Z$,

$x|y$ *and* $y\nmid x$, *yet*

$Z \neq \emptyset$.

1.2 Partial Orders

19 Definition. *P is a* ***partial order*** *iff*

P is a preorder $\langle S, \circ\rangle$, *and*

for each $x \in S$, *for each* $y \in S$,

if $x \circ y$ *and* $y \circ x$ *then* $x = y$.

20 Examples. *The preorder in example 2 is not a partial order, since* $c \circ d$ *and* $d \circ c$, *yet* $c \neq d$. *However, the preorders in examples 3, 4, 5, 6 and 8 are all partial orders.*

21 Proposition. *For each partial order P,*

if P has a top then P has a unique top, and

if P has a bottom then P has a unique bottom.

22 Proposition. *For each partial order* $\langle S, \circ\rangle$, *for each* $Z \subseteq S$,

if Z has a least upper bound in $\langle S, \circ\rangle$
then Z has a unique least upper bound in $\langle S, \circ\rangle$, *and*

if Z has a greatest lower bound in $\langle S, \circ\rangle$
then Z has a unique greatest lower bound in $\langle S, \circ\rangle$.

I therefore speak of *the* top (if it exists), *the* bottom (if it exists), *the* least upper bound (if it exists) of a set, and *the* greatest lower bound (if it exists) of a set in a partial order.

23 Definition. *L is an* ***upper semilattice*** *iff*

L is a partial order $\langle S, \circ\rangle$, *and*

for each $x \in S$, *for each* $y \in S$,

$\{x, y\}$ *has a least upper bound in* $\langle S, \circ\rangle$.

L is a ***lower semilattice*** *iff*

L is a partial order $\langle S, \circ\rangle$, *and*

for each $x \in S$, *for each* $y \in S$,

$\{x, y\}$ *has a greatest lower bound in* $\langle S, \circ\rangle$.

L is a ***lattice*** *iff L is an upper semilattice and L is a lower semilattice.*

24 Examples. *The partial order in example 3 is neither an upper nor a lower semilattice, since* $\{c, d\}$ *no least upper bound and no greatest lower bound. However, the partial orders in examples 4, 5, 6 and 8 are all lattices.*

25 Definition. *For each upper semilattice* $\langle S, \circ\rangle$,

$\vee$ *is the* ***join*** *in* $\langle S, \circ\rangle$ *iff*

$\vee$ *is the total function from* $S \times S$ *to S such that*

for each $x \in S$, *for each* $y \in S$,

$x \vee y$ *is the unique least upper bound of* $\{x, y\}$ *in* $\langle S, \circ\rangle$.

For each lower semilattice $\langle S, \circ\rangle$,

$\wedge$ *is the* ***meet*** *in* $\langle S, \circ\rangle$ *iff*

$\wedge$ *is the total function from* $S \times S$ *to S such that*

for each $x \in S$, *for each* $y \in S$,

$x \wedge y$ *is the unique greatest lower bound of* $\{x, y\}$ *in* $\langle S, \circ\rangle$.

26 Definition. *For each lattice* $\langle S, \circ\rangle$ *with join* $\vee$ *and meet* $\wedge$,

$\langle S, \circ\rangle$ *is* ***distributive*** *iff*

for each $x \in S$, *for each* $y \in S$, *for each* $z \in S$,

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, *and*

$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

27 Examples. *The lattice in example 4 is not distributive, since $f \wedge (e \vee g) = f$, but $(f \wedge e) \vee (f \wedge g) = c$. However, by number theory, the lattice in example 5 is distributive, and, by the de Morgan laws of set theory, the lattices in examples 6 and 8 are distributive.*

28 Definition. *For each lattice $\langle S, \circ \rangle$ with top $\top$, bottom $\bot$, join $\vee$ and meet $\wedge$,*

*x and y are **complements** in $\langle S, \circ \rangle$ iff*

$x \in S$,

$y \in S$,

$x \vee y = \top$, and

$x \wedge y = \bot$.

29 Example. *In the lattice of example 4, c has no complements, whereas e has two complements, d and g.*

30 Proposition. *For each bounded and distributive lattice $\langle S, \circ \rangle$, for each $x \in S$,*

if x has a complement in $\langle S, \circ \rangle$ then x has a unique complement in $\langle S, \circ \rangle$.

I therefore speak of *the* complement (if it exists) of a member of a bounded and distributive lattice.

31 Definition. *B is a **Boolean algebra** iff*

B is a bounded and distributive lattice $\langle S, \circ \rangle$, and

for each $x \in S$, x has a complement in $\langle S, \circ \rangle$.

32 Examples. *The bounded and distributive lattice in example 5 is not a Boolean algebra, since 2 has no complement in $\langle \mathbb{N}, | \rangle$. For each set S, $\langle \mathrm{Pow}(S), \subseteq \rangle$ is a Boolean algebra, regardless of the set S, since for each $X \in \mathrm{Pow}(S)$, $(S \setminus X)$ is the complement of X in $\langle \mathrm{Pow}(S), \subseteq \rangle$. For each preorder $\langle S, \circ \rangle$, $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$ is sometimes a Boolean algebra and sometimes not, contingent on the preorder $\langle S, \circ \rangle$. If $S = \{a\}$ and $a \circ a$ then $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$ is a Boolean algebra. However, if $\langle S, \circ \rangle$ is the preorder in example 3 then $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$ is not a Boolean algebra, since $\{a\}$ has no complement in $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$.*

33 Definition. *For each Boolean algebra $\langle S, \circ \rangle$,*

*$\neg$ is the **complement** in $\langle S, \circ \rangle$ iff*

$\neg$ is the total function from S to S such that

for each $x \in S$, $\neg x$ is the unique complement of x in $\langle S, \circ \rangle$.

34 Definition. *For each lower semilattice $\langle S, \circ \rangle$ with meet $\wedge$,*

*z is a **relative pseudocomplement** of x and y in $\langle S, \circ \rangle$ iff*

$x \in S$,

$y \in S$,

$z \in S$,

$x \wedge z \circ y$, and

for each $w \in S$, if $x \wedge w \circ y$ then $w \circ z$.

35 Proposition. *For each meet semilattice $\langle S, \circ \rangle$, for each $x \in S$, for each $y \in S$,*

if x and y have a relative pseudocomplement in $\langle S, \circ \rangle$

then x and y have a unique relative pseudocomplement in $\langle S, \circ \rangle$.

I therefore speak of *the* relative pseudocomplement (if it exists) of two members of a meet semilattice.

36 Example. *In the lattice of example 4, there is no relative pseudocomplement of e and f, whereas e is the relative pseudocomplement of f and e.*

37 Definition. *For each lower semilattice $\langle S, \circ \rangle$ with bottom $\bot$,*

*z is a **pseudocomplement** of x in $\langle S, \circ \rangle$ iff*

z is a relative pseudocomplement of x and $\bot$ in $\langle S, \circ \rangle$.

38 Proposition. *For each meet semilattice $\langle S, \circ \rangle$ with a bottom, for each $x \in S$,*

if x has a pseudocomplement in $\langle S, \circ \rangle$

then x has a unique pseudocomplement in $\langle S, \circ \rangle$.

I therefore speak of *the* pseudocomplement (if it exists) of a member of a meet semilattice with a bottom.

39 Example. *In the lattice of example 4, g is the pseudocomplement of b, and e is the pseudocomplement of g.*

40 Definition. *H is a **Heyting algebra** iff*

H is a bounded and distributive lattice $\langle S, \circ \rangle$, and

for each $x \in S$, for each $y \in S$,

x and y have a relative pseudocomplement in $\langle S, \circ \rangle$.

41 Proposition. *Each Boolean algebra is a Heyting algebra.*

42 Examples. *The bounded and distributive lattice in example 5 is not a Heyting algebra, since 2 and 3 has no relative pseudocomplement in* $\langle \mathbb{N}, | \rangle$. *For each set* S, $\langle \mathrm{Pow}(S), \subseteq \rangle$ *is a Heyting algebra, regardless of the set* S, *since for each* $X \in \mathrm{Pow}(S)$, *for each* $Y \in \mathrm{Pow}(S)$, $(S \setminus X) \cup Y$ *is the relative pseudocomplement of* X *and* Y *in* $\langle \mathrm{Pow}(S), \circ \rangle$. *For each preorder* $\langle S, \circ \rangle$, $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$ *is also a Heyting algebra, regardless of the preorder* $\langle S, \circ \rangle$, *since for each* $X \in \mathrm{Down}(\langle S, \circ \rangle)$, *for each* $Y \in \mathrm{Down}(\langle S, \circ \rangle)$, *the relative pseudocomplement of* X *and* Y *in* $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$ *is* $\bigcup \{Z \in \mathrm{Down}(\langle S, \circ \rangle) \mid X \cap Z \subseteq Y\}$. *For example, if* $\langle S, \circ \rangle$ *is the preorder in example 3 then the relative pseudocomplement of* $\{a\}$ *and* $\{b\}$ *in* $\langle \mathrm{Down}(\langle S, \circ \rangle), \subseteq \rangle$ *is* $\{b\}$.

43 Definition. *For each Heyting algebra* $\langle S, \circ \rangle$,

$\to$ *is the* ***relative pseudocomplement*** *in* $\langle S, \circ \rangle$ *iff*
- $\to$ *is the total function from* $S \times S$ *to* S *such that*
  - *for each* $x \in S$, *for each* $y \in S$,
    - $x \to y$ *is the unique relative pseudocomplement of* x *and* y *in* $\langle X, \circ \rangle$, *and*

$\neg$ *is the* ***pseudocomplement*** *in* $\langle S, \circ \rangle$ *iff*
- $\neg$ *is the total function from* S *to* S *such that*
  - *for each* $x \in S$, $\neg x$ *is the unique pseudocomplement of* x *in* $\langle X, \circ \rangle$.

1.3 Equivalence Relations

44 Definition. E *is an* ***equivalence relation*** *iff*
- E *is a preorder* $\langle S, \circ \rangle$, *and*
- *for each* $x \in S$, *for each* $y \in S$,
  - *if* $x \circ y$ *then* $y \circ x$.

45 Example. *If* $\mathbb{N}$ *is the set of natural numbers and* $n_1 \wr n_2$ *iff* $n_1 + n_2$ *is even then* $\langle \mathbb{N}, \wr \rangle$ *is an equivalence relation.*

46 Definition. *For each equivalence relation* $\langle S, \circ \rangle$,

C *is an* ***equivalence class*** *of* $\langle S, \circ \rangle$ *iff*
- $C \subseteq S$,
- $C \neq \emptyset$, *and*
- *for each* $x \in C$, *for each* $y \in S$,
  - $y \in C$ *iff* $x \circ y$.

If $\langle S, \circ \rangle$ is an equivalence relation then I write $\mathrm{Quo}(\langle S, \circ \rangle)$ for the set of all equivalence classes of $\langle S, \circ \rangle$, and call $\mathrm{Quo}(\langle S, \circ \rangle)$ the **quotient** of $\langle S, \circ \rangle$.

47 Example. *The quotient of the equivalence relation in example 45 comprises two sets, the set of even natural numbers and the set of odd natural numbers.*

48 Proposition. *For each equivalence relation* $\langle S, \circ \rangle$, *for each* $x \in S$,
- *for some unique* $C \in \mathrm{Quo}(\langle S, \circ \rangle)$, $x \in C$

If $\langle S, \circ \rangle$ is an equivalence relation and $x \in S$ then I write $|x|_{\langle S, \circ \rangle}$ for the unique equivalence class of $\langle S, \circ \rangle$ containing x. Proposition 48 has a number of corollaries.

49 Corollary. *For each equivalence relation* $\langle S, \circ \rangle$,
- $S = \bigcup \mathrm{Quo}(\langle S, \circ \rangle)$,
- *for each* $C_1 \in \mathrm{Quo}(\langle S, \circ \rangle)$, *for each* $C_2 \in \mathrm{Quo}(\langle S, \circ \rangle)$,
  - $C_1 = C_2$ *or* $C_1 \cap C_2 = \emptyset$, *and*
- *for each* $x \in S$, *for each* $y \in S$,
  - $x \circ y$ *iff* $|x|_{\langle S, \circ \rangle} = |y|_{\langle S, \circ \rangle}$.

Thus, the equivalence relation $\langle \mathrm{Quo}(\langle S, \circ \rangle), = \rangle$ is a 'distillation' of the equivalence relation $\langle S, \circ \rangle$ that reduces '$\circ$' to '$=$'.

2 Feature Structures

2.1 Signatures

50 Definition. Sign *is a* ***signature*** *iff*

Sign *is a triple* $\langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$,

$\langle \mathrm{Sort}, \trianglelefteq \rangle$ *is a partial order, and*

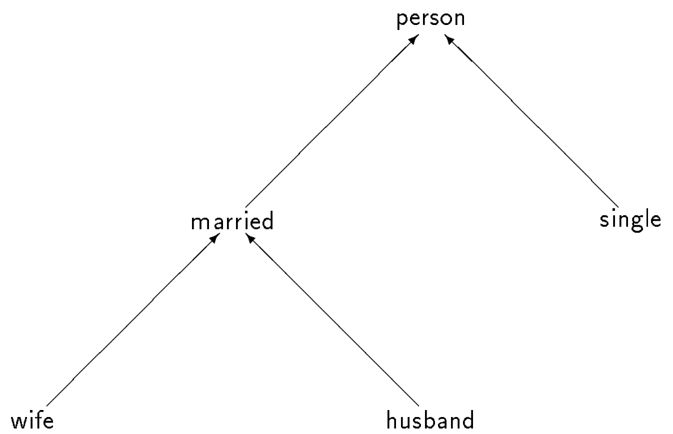
Feat *is a set.*

If Sign $= \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$ is a signature then I call each member of Sort a **sort** under Sign, and each member of Feat a **feature** under Sign, and read $\sigma_1 \trianglelefteq \sigma_2$ as sort σ_2 **subsumes** sort σ_1 under Sign.

51 Example. *If*

Sort $= \{\mathsf{person}, \mathsf{married}, \mathsf{single}, \mathsf{wife}, \mathsf{husband}\}$,

$x \trianglelefteq y$ *iff there is a sequence (possibly empty) of arrows from* x *to* y *in*



Feat $= \{\textsc{spouse}, \textsc{friend}\}$, *and*

Sign $= \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$

then Sign *is a signature.*

2.2 Concrete Feature Structures

52 Definition. *For each signature* Sign $= \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$,

$\mathfrak{C}$ *is a* ***concrete feature structure*** *under* Sign *iff*

$\mathfrak{C}$ *is a quintuple* $\langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R} \rangle$,

$\mathcal{N}$ *is a set,*

$\hat{\nu} \in \mathcal{N}$,

$\mathcal{A}$ *is a partial function from* $\mathcal{N} \times \mathrm{Feat}$ *to* $\mathcal{N}$,

$\mathcal{L}$ *is a total function from* $\mathcal{N}$ *to* Sort,

for each $\nu \in \mathcal{N}$, *for some natural number* i, *for some* $\nu_0 \in \mathcal{N}$, …, *for some* $\nu_i \in \mathcal{N}$,

$\nu_0 = \hat{\nu}$,

for each natural number $j < i$, *for some* $\varphi \in \mathrm{Feat}$,

$\mathcal{A}\langle \nu_j, \varphi \rangle$ *is defined and* $\mathcal{A}\langle \nu_j, \varphi \rangle = \nu_{j+1}$, *and*

$\nu_i = \nu$,

$\mathcal{R}$ *is a binary relation on* $\mathcal{N}$,

for each $\nu \in \mathcal{N}$, $\langle \nu, \nu \rangle \notin \mathcal{R}$, *and*

for each $\nu_1 \in \mathcal{N}$, *for each* $\nu_2 \in \mathcal{N}$,

if $\langle \nu_1, \nu_2 \rangle \in \mathcal{R}$ *then* $\langle \nu_2, \nu_1 \rangle \in \mathcal{R}$.

If Sign is a signature then I write $\mathrm{CFS}_{\mathrm{Sign}}$ for the class of concrete feature structures under Sign.

53 Example. *If*

Sign *is the signature of example 51,*

$\mathcal{N} = \{\hat{\nu}, \nu\}$,

$\mathcal{A}$ *is the partial function from* $\mathcal{N} \times \mathrm{Feat}$ *to* $\mathcal{N}$ *such that*

$\mathcal{A}\langle \hat{\nu}, \textsc{spouse} \rangle = \nu$, *and*

$\mathcal{A}\langle \hat{\nu}, \textsc{friend} \rangle = \nu$,

$\mathcal{L}$ is the total function from $\mathcal{N}$ to Sort *such that*

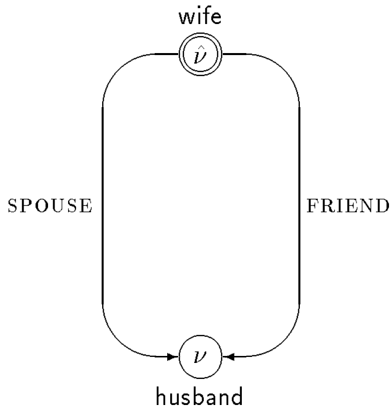
$\mathcal{L}(\hat{\nu}) = \mathsf{wife}$, *and*

$\mathcal{L}(\nu) = \mathsf{husband}$,

$\mathcal{R} = \emptyset$, *and*

$\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R} \rangle$

then $\mathfrak{C}$ is a concrete feature structure under Sign*. I depict $\mathfrak{C}$ as*



54 Example. *If*

Sign *is the signature of example 51,*

$\mathcal{N} = \{\hat{\nu}, \nu\}$,

$\mathcal{A}$ is the partial function from $\mathcal{N} \times$ Feat *to $\mathcal{N}$ such that*

$\mathcal{A}\langle \hat{\nu}, \text{SPOUSE} \rangle = \nu$, *and*

$\mathcal{A}\langle \nu, \text{SPOUSE} \rangle = \hat{\nu}$,

$\mathcal{L}$ is the total function from $\mathcal{N}$ to Sort *such that*

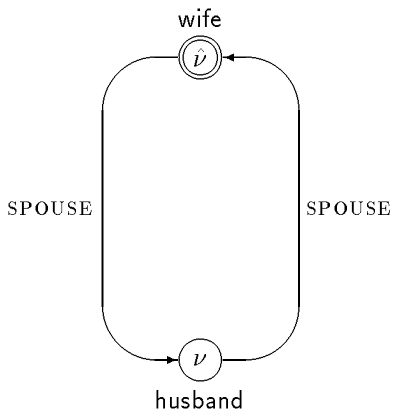
$\mathcal{L}(\hat{\nu}) = \mathsf{wife}$, *and*

$\mathcal{L}(\nu) = \mathsf{husband}$,

$\mathcal{R} = \emptyset$, *and*

$\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R} \rangle$

then $\mathfrak{C}$ is a concrete feature structure under Sign*. I depict $\mathfrak{C}$ as*



55 Example. *If*

Sign *is the signature of example 51,*

$\mathcal{N} = \{\hat{\nu}, \nu_1, \nu_2\}$,

$\mathcal{A}$ is the partial function from $\mathcal{N} \times$ Feat *to $\mathcal{N}$ such that*

$\mathcal{A}\langle \hat{\nu}, \text{SPOUSE} \rangle = \nu_1$, *and*

$\mathcal{A}\langle \hat{\nu}, \text{FRIEND} \rangle = \nu_2$,

$\mathcal{L}$ is the total function from $\mathcal{N}$ to Sort *such that*

$\mathcal{L}(\hat{\nu}) = \mathsf{married}$,

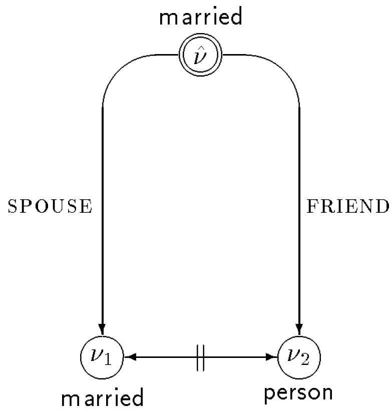
$\mathcal{L}(\nu_1) = \mathsf{married}$, *and*

$\mathcal{L}(\nu_2) = \mathsf{person}$,

$\mathcal{R} = \{\langle \nu_1, \nu_2 \rangle, \langle \nu_2, \nu_1 \rangle\}$, *and*

$\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R} \rangle$

then $\mathfrak{C}$ *is a concrete feature structure under* Sign. *I depict* $\mathfrak{C}$ *as*



56 Definition. *For each signature* $\text{Sign} = \langle \text{Sort}, \trianglelefteq, \text{Feat} \rangle$*, for each* $\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R} \rangle \in \text{CFS}_{\text{Sign}}$,

$\mathcal{A}^*$ *is the partial function from* $\mathcal{N} \times \text{Feat}^*$ *to* $\mathcal{N}$ *such that,*

for each $\nu \in \mathcal{N}$,

$\mathcal{A}^*\langle \nu, \varepsilon \rangle$ *is defined, and*

$\mathcal{A}^*\langle \nu, \varepsilon \rangle = \nu$*, and*

for each $\nu \in \mathcal{N}$*, for each* $\pi \in \text{Feat}^*$*, for each* $\varphi \in \text{Feat}$,

$\mathcal{A}^*\langle \nu, \pi\varphi \rangle$ *is defined iff* $\mathcal{A}^*\langle \nu, \pi \rangle$ *is defined and* $\mathcal{A}\langle \mathcal{A}^*\langle \nu, \pi \rangle, \varphi \rangle$ *is defined, and*

if $\mathcal{A}^*\langle \nu, \pi\varphi \rangle$ *is defined then* $\mathcal{A}^*\langle \nu, \pi\varphi \rangle = \mathcal{A}\langle \mathcal{A}^*\langle \nu, \pi \rangle, \varphi \rangle$.

57 Example. *If*

Sign *is the signature of example 51, and*

$\mathfrak{C}$ *is the concrete feature structure under* Sign *of example 54*

then

$\mathcal{A}^*\langle \hat{\nu}, \varepsilon \rangle = \hat{\nu}$,[2]

$\mathcal{A}^*\langle \hat{\nu}, \text{SPOUSE} \rangle = \nu$,

$\mathcal{A}^*\langle \hat{\nu}, \text{FRIEND} \rangle$ *is undefined,*

$\mathcal{A}^*\langle \hat{\nu}, \text{SPOUSE SPOUSE} \rangle = \hat{\nu}$,

$\mathcal{A}^*\langle \hat{\nu}, \text{SPOUSE FRIEND} \rangle$ *is undefined,*

$\mathcal{A}^*\langle \hat{\nu}, \text{FRIEND SPOUSE} \rangle$ *is undefined,*

$\mathcal{A}^*\langle \hat{\nu}, \text{FRIEND FRIEND} \rangle$ *is undefined,*

$\mathcal{A}^*\langle \hat{\nu}, \text{SPOUSE SPOUSE SPOUSE} \rangle = \nu$,

$\mathcal{A}^*\langle \hat{\nu}, \text{SPOUSE SPOUSE FRIEND} \rangle$ *is undefined,*

$\mathcal{A}^*\langle \hat{\nu}, \text{SPOUSE FRIEND SPOUSE} \rangle$ *is undefined,*

$\mathcal{A}^*\langle \hat{\nu}, \text{SPOUSE FRIEND FRIEND} \rangle$ *is undefined,*

etc.

58 Definition. *For each signature* $\text{Sign} = \langle \text{Sort}, \trianglelefteq, \text{Feat} \rangle$*, for each* $\mathfrak{C}_1 = \langle \mathcal{N}_1, \hat{\nu}_1, \mathcal{A}_1, \mathcal{L}_1, \mathcal{R}_1 \rangle \in \text{CFS}_{\text{Sign}}$*, for each* $\mathfrak{C}_2 = \langle \mathcal{N}_2, \hat{\nu}_2, \mathcal{A}_2, \mathcal{L}_2, \mathcal{R}_2 \rangle \in \text{CFS}_{\text{Sign}}$,

μ *is a* ***morphism*** *from* $\mathfrak{C}_1$ *to* $\mathfrak{C}_2$ *under* Sign *iff*

μ *is a total function from* $\mathcal{N}_1$ *to* $\mathcal{N}_2$,

$\mu(\hat{\nu}_1) = \hat{\nu}_2$,

for each $\nu \in \mathcal{N}_1$*, for each* $\varphi \in \text{Feat}$,

if $\mathcal{A}_1\langle \nu, \varphi \rangle$ *is defined*

then $\mathcal{A}_2\langle \mu(\nu), \varphi \rangle$ *is defined and* $\mathcal{A}_2\langle \mu(\nu), \varphi \rangle = \mu(\mathcal{A}_1\langle \nu, \varphi \rangle)$,

for each $\nu \in \mathcal{N}_1$, $\mathcal{L}_2(\mu(\nu)) \trianglelefteq \mathcal{L}_1(\nu)$*, and*

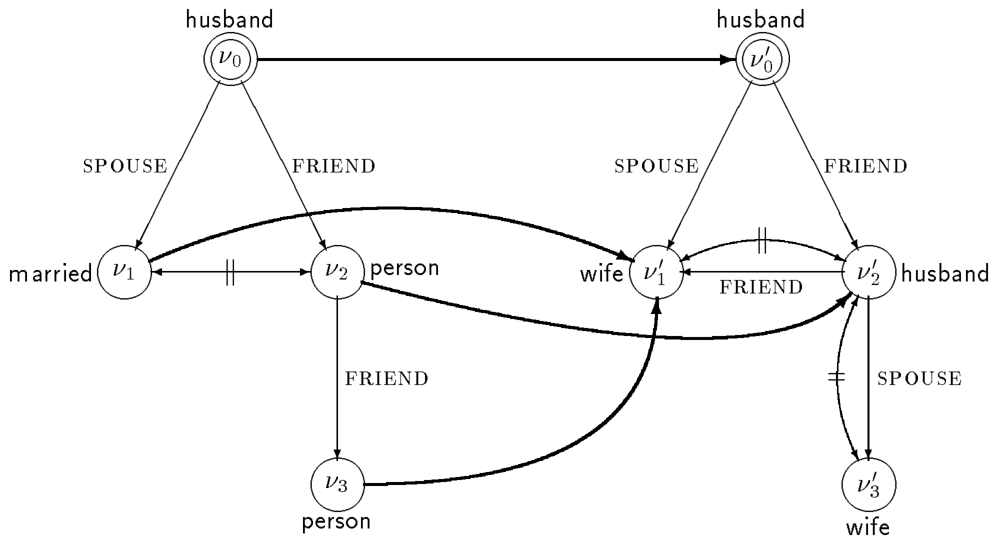
for each $\nu_1 \in \mathcal{N}_1$*, for each* $\nu_2 \in \mathcal{N}_1$,

if $\langle \nu_1, \nu_2 \rangle \in \mathcal{R}_1$ *then* $\langle \mu(\nu_1), \mu(\nu_2) \rangle \in \mathcal{R}_2$.

59 Example. *The thick arrows in the following diagram depict a morphism from the concrete feature structure on the left to the concrete feature on the right under the signature of example 51.*

---

[2] I write ε for the empty string.



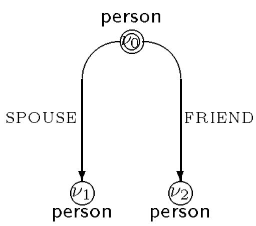
60 Definition. *For each signature* Sign, *for each* $\mathfrak{C}_1 \in \mathrm{CFS}_{\mathrm{Sign}}$, *for each* $\mathfrak{C}_2 \in \mathrm{CFS}_{\mathrm{Sign}}$,

$\mathfrak{C}_1$ ***subsumes*** $\mathfrak{C}_2$ *under* Sign *iff*

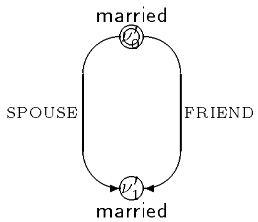
for some μ, μ *is a morphism from* $\mathfrak{C}_1$ *to* $\mathfrak{C}_2$ *under* Sign.

If Sign is a signature then I write $\mathfrak{C}_1 \preceq_{\mathrm{Sign}} \mathfrak{C}_2$ for concrete feature structure $\mathfrak{C}_2$ subsumes concrete feature structure $\mathfrak{C}_1$ under Sign.

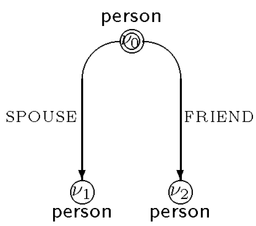
61 Example. *Under the signature of example 51,*



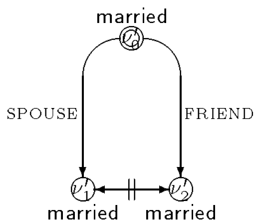
subsumes



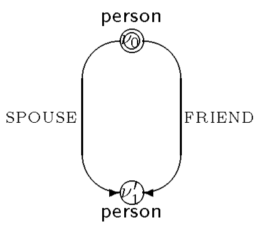
, *and*



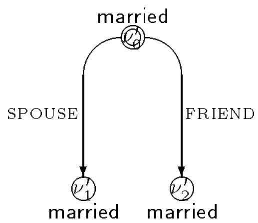
subsumes



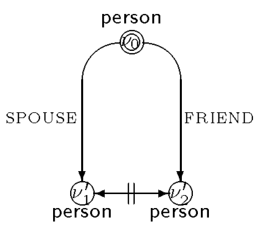
, *but*



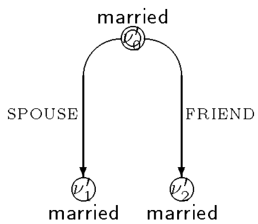
does not subsume



, *and*



does not subsume



.

62 Proposition. *For each signature* Sign,

for each $\mathfrak{C} \in \mathrm{CFS}_{\mathrm{Sign}}$,

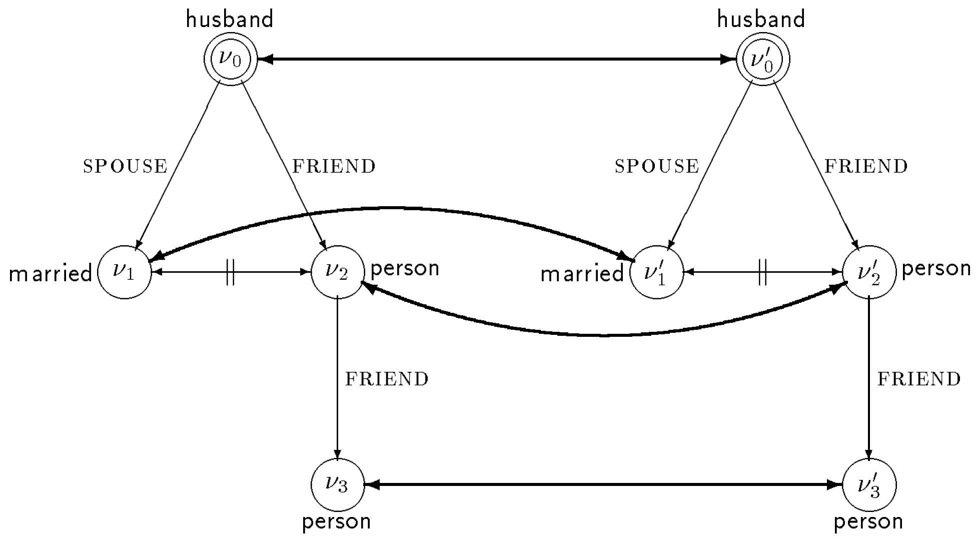
$\mathfrak{C} \preceq_{\mathrm{Sign}} \mathfrak{C}$, *and*

for each $\mathfrak{C}_1 \in \mathrm{CFS}_{\mathrm{Sign}}$, *for each* $\mathfrak{C}_2 \in \mathrm{CFS}_{\mathrm{Sign}}$, *for each* $\mathfrak{C}_3 \in \mathrm{CFS}_{\mathrm{Sign}}$,

if $\mathfrak{C}_1 \preceq_{\mathrm{Sign}} \mathfrak{C}_2$ *and* $\mathfrak{C}_2 \preceq_{\mathrm{Sign}} \mathfrak{C}_3$ *then* $\mathfrak{C}_1 \preceq_{\mathrm{Sign}} \mathfrak{C}_3$.

If Sign is a signature then $\langle \mathrm{CFS}_{\mathrm{Sign}}, \preceq_{\mathrm{Sign}} \rangle$ does not constitute a preorder, despite proposition 62. This is on the rather technical (and circumventable) grounds that $\mathrm{CFS}_{\mathrm{Sign}}$ may be a *proper class*, not a set. Nonetheless, even if this objection is put aside, the next example shows that $\langle \mathrm{CFS}_{\mathrm{Sign}}, \preceq_{\mathrm{Sign}} \rangle$ may not constitute a partial order.

63 Example. *If* Sign *is the signature of example 51 then the thick double arrows in the following diagram depict both a morphism from the concrete feature structure on the left to the concrete feature on the right and a morphism from the concrete feature structure on the right to the concrete feature structure on the left under* Sign*. Consequently, each of the two concrete feature structures subsumes the other under* Sign*, yet the two concrete feature structures are not identical.*



2.3 Abstract Feature Structures

Most HPSG formalisms that use feature structures require least upper bounds and greatest lower bounds to be unique. Consequently, by proposition 22, it is desirable that the set of feature structures and the subsumption relation under a signature constitute a partial order. Fortunately, there are a number of ways of drawing a (true) partial order of features structures out of a (quasi) preorder of concrete feature structures. In these notes, I extend an extremely elegant method due to [Moshier 1988].

64 Definition. *For each signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$,

$\mathfrak{A}$ *is an* ***abstract feature structure*** *under* Sign *iff*

$\mathfrak{A}$ *is a quadruple* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle$,

$\mathcal{D} \subseteq \mathrm{Feat}^*$,

$\varepsilon \in \mathcal{D}$,

for each $\pi \in \mathrm{Feat}^*$, *for each* $\varphi \in \mathrm{Feat}$,

if $\pi\varphi \in \mathcal{D}$ *then* $\pi \in \mathcal{D}$,

$\langle \mathcal{D}, \mathcal{E} \rangle$ *is an equivalence relation,*

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$, *for each* $\varphi \in \mathrm{Feat}$,

if $\pi_1\varphi \in \mathcal{D}$ *and* $\langle \pi_1, \pi_2 \rangle \in \mathcal{E}$ *then* $\pi_2\varphi \in \mathcal{D}$ *and* $\langle \pi_1\varphi, \pi_2\varphi \rangle \in \mathcal{E}$,

$\mathcal{F}$ *is a total function from* $\mathcal{D}$ *to* Sort,

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

if $\langle \pi_1, \pi_2 \rangle \in \mathcal{E}$ *then* $\mathcal{F}(\pi_1) = \mathcal{F}(\pi_2)$,

$\mathcal{I} \subseteq \mathcal{D} \times \mathcal{D}$,

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

if $\langle \pi_1, \pi_2 \rangle \in \mathcal{I}$ *then* $\langle \pi_2, \pi_1 \rangle \in \mathcal{I}$,

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

if $\langle \pi_1, \pi_2 \rangle \in \mathcal{I}$ *then* $\langle \pi_1, \pi_2 \rangle \notin \mathcal{E}$, *and*

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$, *for each* $\pi_3 \in \mathrm{Feat}^*$,

if $\langle \pi_1, \pi_2 \rangle \in \mathcal{I}$ *and* $\langle \pi_2, \pi_3 \rangle \in \mathcal{E}$ *then* $\langle \pi_1, \pi_3 \rangle \in \mathcal{I}$.

If Sign is a signature then I write $\mathrm{AFS}_{\mathrm{Sign}}$ for the set of abstract feature structures under Sign.

65 Definition. *For each signature* Sign $= \langle \text{Sort}, \trianglelefteq, \text{Feat}\rangle$, *for each* $\mathfrak{A}_1 = \langle \mathcal{D}_1, \mathcal{E}_1, \mathcal{F}_1, \mathcal{I}_1\rangle \in \text{AFS}_{\text{Sign}}$, *for each* $\mathfrak{A}_2 = \langle \mathcal{D}_2, \mathcal{E}_2, \mathcal{F}_2, \mathcal{I}_2\rangle \in \text{AFS}_{\text{Sign}}$,

$\mathfrak{A}_1$ ***subsumes*** $\mathfrak{A}_2$ *under* Sign *iff*

$\mathcal{D}_1 \subseteq \mathcal{D}_2$,

$\mathcal{E}_1 \subseteq \mathcal{E}_2$,

for each $\pi \in \mathcal{D}_1$, $\mathcal{F}_2(\pi) \trianglelefteq \mathcal{F}_1(\pi)$, *and*

$\mathcal{R}_1 \subseteq \mathcal{R}_2$.

If Sign is a signature then I write $\mathfrak{A}_1 \sqsubseteq_{\text{Sign}} \mathfrak{A}_2$ for abstract feature structure $\mathfrak{A}_2$ subsumes abstract feature structure $\mathfrak{A}_1$ under Sign.

66 Proposition. *For each signature* Sign, $\langle \text{AFS}_{\text{Sign}}, \sqsubseteq_{\text{Sign}}\rangle$ *is a partial order.*

67 Definition. *For each signature* Sign $= \langle \text{Sort}, \trianglelefteq, \text{Feat}\rangle$, *for each* $\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R}\rangle \in \text{CFS}_{\text{Sign}}$,

$\text{Abst}_{\text{Sign}}(\mathfrak{C})$ *is the quadruple* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle$, *where*

$$\mathcal{D} = \left\{\pi \in \text{Feat}^* \,\middle|\, \mathcal{A}^*\langle \hat{\nu}, \pi\rangle \text{ is defined}\right\},$$

$$\mathcal{E} = \left\{ \langle \pi_1, \pi_2\rangle \in \text{Feat}^* \times \text{Feat}^* \,\middle|\, \begin{array}{l} \mathcal{A}^*\langle \hat{\nu}, \pi_1\rangle \text{ is defined,} \\ \mathcal{A}^*\langle \hat{\nu}, \pi_2\rangle \text{ is defined, and} \\ \mathcal{A}^*\langle \hat{\nu}, \pi_1\rangle = \mathcal{A}^*\langle \hat{\nu}, \pi_2\rangle \end{array} \right\},$$

$$\mathcal{F} = \left\{ \langle \pi, \sigma\rangle \in \text{Feat}^* \times \text{Sort} \,\middle|\, \begin{array}{l} \mathcal{A}^*\langle \hat{\nu}, \pi\rangle \text{ is defined, and} \\ \mathcal{L}(\mathcal{A}^*\langle \hat{\nu}, \pi\rangle) = \sigma \end{array} \right\}, \text{ and}$$

$$\mathcal{I} = \left\{ \langle \pi_1, \pi_2\rangle \in \text{Feat}^* \times \text{Feat}^* \,\middle|\, \begin{array}{l} \mathcal{A}^*\langle \hat{\nu}, \pi_1\rangle \text{ is defined,} \\ \mathcal{A}^*\langle \hat{\nu}, \pi_2\rangle \text{ is defined, and} \\ \langle \mathcal{A}^*\langle \hat{\nu}, \pi_1\rangle, \mathcal{A}^*\langle \hat{\nu}, \pi_2\rangle\rangle \in \mathcal{R} \end{array} \right\}.$$

68 Proposition. *For each signature* Sign,

for each $\mathfrak{C} \in \text{CFS}_{\text{Sign}}$, *for some* $\mathfrak{A} \in \text{AFS}_{\text{Sign}}$, $\text{Abst}_{\text{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{A} \in \text{AFS}_{\text{Sign}}$, *for some* $\mathfrak{C} \in \text{CFS}_{\text{Sign}}$, $\text{Abst}_{\text{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{C}_1 \in \text{CFS}_{\text{Sign}}$, *for each* $\mathfrak{C}_2 \in \text{CFS}_{\text{Sign}}$,

$\mathfrak{C}_1 \preceq_{\text{Sign}} \mathfrak{C}_2$ *iff* $\text{Abst}_{\text{Sign}}(\mathfrak{C}_1) \sqsubseteq_{\text{Sign}} \text{Abst}_{\text{Sign}}(\mathfrak{C}_2)$, *and*

for each $\mathfrak{C}_1 \in \text{CFS}_{\text{Sign}}$, *for each* $\mathfrak{C}_2 \in \text{CFS}_{\text{Sign}}$,

$\mathfrak{C}_1 \preceq_{\text{Sign}} \mathfrak{C}_2$ *and* $\mathfrak{C}_2 \preceq_{\text{Sign}} \mathfrak{C}_1$ *iff* $\text{Abst}_{\text{Sign}}(\mathfrak{C}_1) = \text{Abst}_{\text{Sign}}(\mathfrak{C}_2)$.

If Sign is a signature then I call $\text{Abst}_{\text{Sign}}$ the **abstraction function** under Sign. Proposition 68 shows that we can safely use the abstraction function to move from a class sized preorder of concrete feature structures to a set sized partial order of abstract feature structures.

69 Example. *If*

Sign *is the signature of example 51, and*

$\mathfrak{C}$ *is the concrete feature structure under* Sign *of example 54*

then $\text{Abst}_{\text{Sign}}(\mathfrak{C}) = \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle$, *where*

$\mathcal{D} = \left\{\text{SPOUSE}^i \,\middle|\, i \text{ is a natural number}\right\}$,

$\mathcal{E} = \left\{\langle \text{SPOUSE}^i, \text{SPOUSE}^j\rangle \,\middle|\, i \text{ and } j \text{ are even natural numbers}\right\}$

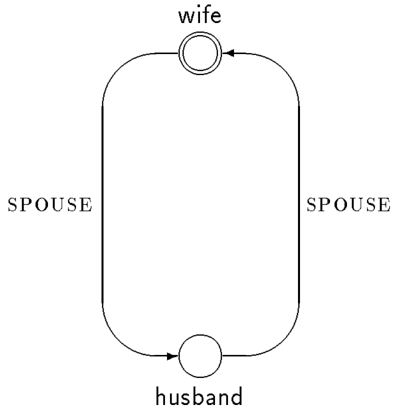
$\cup \left\{\langle \text{SPOUSE}^i, \text{SPOUSE}^j\rangle \,\middle|\, i \text{ and } j \text{ are odd natural numbers}\right\}$,

$\mathcal{F} = \left\{\langle \text{SPOUSE}^i, \mathsf{wife}\rangle \,\middle|\, i \text{ is an even natural number}\right\}$

$\cup \left\{\langle \text{SPOUSE}^i, \mathsf{husband}\rangle \,\middle|\, i \text{ is an odd natural number}\right\}$, *and*

$\mathcal{I} = \emptyset$.

I depict $\text{Abst}_{\text{Sign}}(\mathfrak{C})$ *as*



70 Example. *If*

  Sign *is the signature of example 51, and*

  $\mathfrak{C}$ *is the concrete feature structure under* Sign *of example 55*

then $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}) = \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle$, *where*

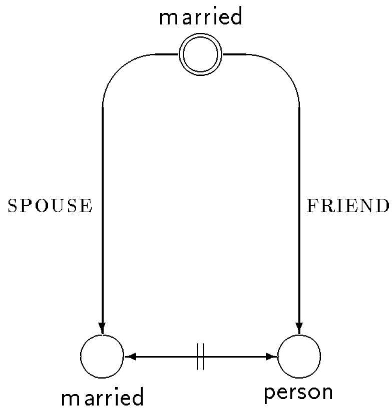
  $\mathcal{D} = \{\varepsilon, \text{SPOUSE}, \text{FRIEND}\}$,

  $\mathcal{E} = \{\langle \varepsilon, \varepsilon \rangle, \langle \text{SPOUSE}, \text{SPOUSE} \rangle, \langle \text{FRIEND}, \text{FRIEND} \rangle\}$,

  $\mathcal{F} = \{\langle \varepsilon, \mathsf{married} \rangle, \langle \text{SPOUSE}, \mathsf{married} \rangle, \langle \text{FRIEND}, \mathsf{person} \rangle\}$, *and*

  $\mathcal{I} = \{\langle \text{SPOUSE}, \text{FRIEND} \rangle, \langle \text{FRIEND}, \text{SPOUSE} \rangle\}$.

I depict $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C})$ *as*



71 Definition. *For each signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$, *for each* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{AFS}_{\mathrm{Sign}}$, *for each* $\pi \in \mathrm{Feat}^*$,

  $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle @\pi = \langle \mathcal{D}', \mathcal{E}', \mathcal{F}', \mathcal{I}' \rangle$, *where*

    $\mathcal{D}' = \left\{ \pi' \in \mathrm{Feat}^* \,\middle|\, \pi\pi' \in \mathcal{D} \right\}$,

    $\mathcal{E}' = \left\{ \langle \pi_1, \pi_2 \rangle \in \mathrm{Feat}^* \times \mathrm{Feat}^* \,\middle|\, \langle \pi\pi_1, \pi\pi_2 \rangle \in \mathcal{E} \right\}$,

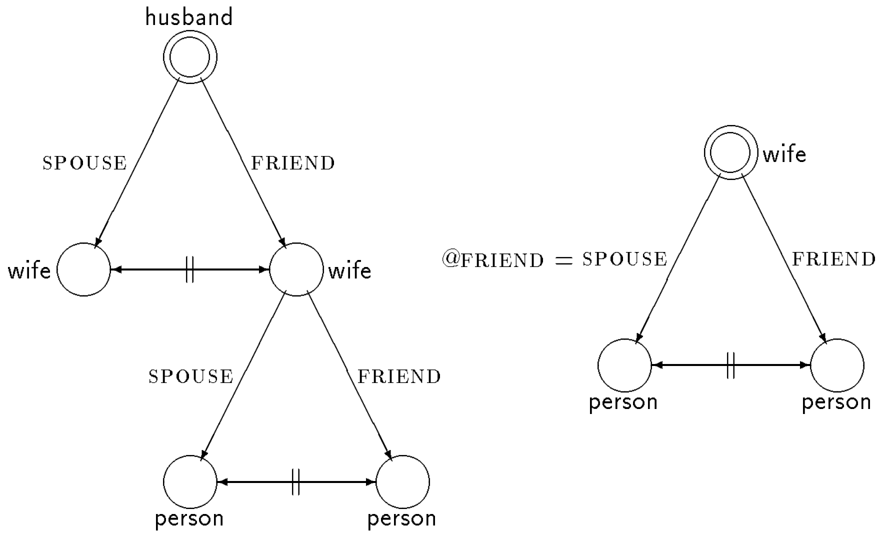
    $\mathcal{F}' = \left\{ \langle \pi', \sigma \rangle \in \mathrm{Feat}^* \times \mathrm{Sort} \,\middle|\, \langle \pi\pi', \sigma \rangle \in \mathcal{F} \right\}$, *and*

    $\mathcal{I}' = \left\{ \langle \pi_1, \pi_2 \rangle \in \mathrm{Feat}^* \times \mathrm{Feat}^* \,\middle|\, \langle \pi\pi_1, \pi\pi_2 \rangle \in \mathcal{I} \right\}$.

72 Proposition. *For each signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$, *for each* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{AFS}_{\mathrm{Sign}}$, *for each* $\pi \in \mathrm{Feat}^*$,

  if $\pi \in \mathcal{D}$ *then* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle @\pi \in \mathrm{AFS}_{\mathrm{Sign}}$.

73 Example. *If* Sign *is the signature of example 51 then*



2.4 Disjunctive Feature Structures

74 Definition. *For each signature* Sign,

  $\mathfrak{D}$ *is a **disjunctive feature structure** under* Sign *iff*

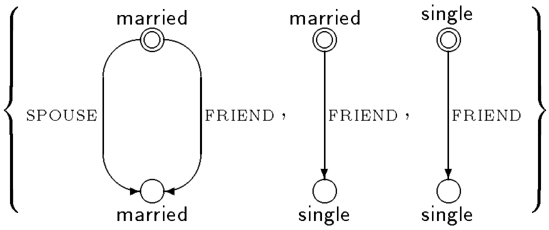
    $\mathfrak{D} \subseteq \mathrm{AFS}_{\mathrm{Sign}}$, *and*

    for each $\mathfrak{A}_1 \in \mathfrak{D}$, *for each* $\mathfrak{A}_2 \in \mathfrak{D}$,

      if $\mathfrak{A}_1 \sqsubseteq_{\mathrm{Sign}} \mathfrak{A}_2$ *then* $\mathfrak{A}_1 = \mathfrak{A}_2$.

If Sign is a signature then I write $\mathrm{DFS}_{\mathrm{Sign}}$ for the set of disjunctive feature structures under Sign.

75 Example.



depicts a disjunctive feature structure under the signature of example 51.

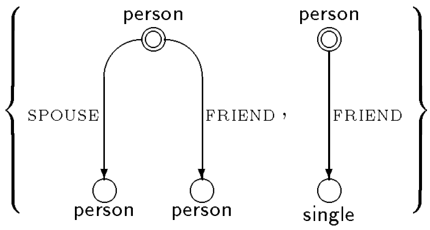
76 Definition. *For each signature* Sign, *for each* $\mathfrak{D}_1 \in \mathrm{DFS}_{\mathrm{Sign}}$, *for each* $\mathfrak{D}_2 \in \mathrm{DFS}_{\mathrm{Sign}}$,

$\mathfrak{D}_1$ *subsumes* $\mathfrak{D}_2$ *under* Sign

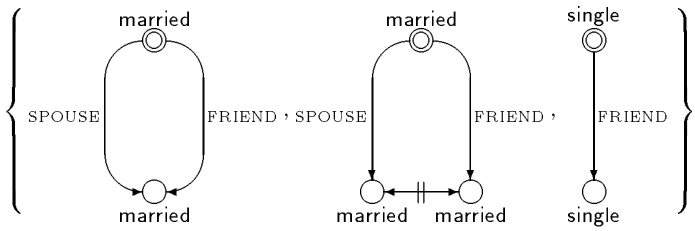
iff for each $\mathfrak{A}_2 \in \mathfrak{D}_2$, *for some* $\mathfrak{A}_1 \in \mathfrak{D}_1$, $\mathfrak{A}_2 \sqsubseteq_{\mathrm{Sign}} \mathfrak{A}_1$.

If Sign is a signature then I write $\mathfrak{D}_1 \ll_{\mathrm{Sign}} \mathfrak{D}_2$ for disjunctive feature structure $\mathfrak{D}_2$ subsumes disjunctive feature structure $\mathfrak{D}_1$ under Sign.

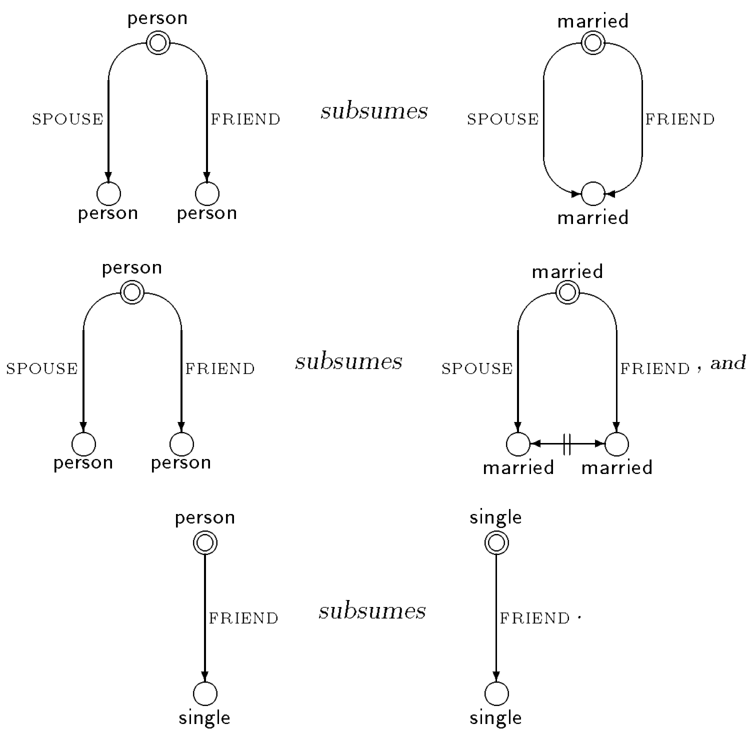
77 Example. *Under the signature of example 51, disjunctive feature structure*



subsumes disjunctive feature structure



since



78 Proposition. *For each signature* Sign, $\langle \mathrm{DFS}_{\mathrm{Sign}}, \ll_{\mathrm{Sign}} \rangle$ *is a partial order.*

3 The [Pollard and Sag 1987] Formalism

3.1 The Formalism

The [Pollard and Sag 1987] formalism can be summarised by the following slogans:

A natural language can be identified with partial information held in common by all mature users of the natural language.

Partial information can be modelled by a certain kind of disjunctive feature structure.

The disjunctive feature structures constitute a Heyting algebra.

A disjunctive feature structure can be constructed from given abstract feature structures using the operations of the Heyting algebra.

Thus, for the [Pollard and Sag 1987] formalism, a grammar of a natural language is essentially a recipe for using the operations of a Heyting algebra to construct from given abstract feature structures a disjunctive feature structure that models the partial information held in common by all mature users of the natural language.

79 Definition. Sign *is a* ***PS87 signature*** *iff*

Sign *is a signature* $\langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}\rangle$*, and*

$\langle \mathrm{Sort}, \trianglelefteq\rangle$ *is upward Noetherian.*

80 Example. *The signature of example 51 is a PS87 signature.*

81 Definition. *For each PS87 signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}\rangle$,

$\mathfrak{C}$ *is a* ***PS87 concrete feature structure*** *under* Sign *iff*

$\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R}\rangle \in \mathrm{CFS}_{\mathrm{Sign}}$

$\mathcal{A}$ *is finite,*

for each $\nu \in \mathcal{N}$, *for each* $\pi \in \mathrm{Feat}^*$,

if $\mathcal{A}^*\langle \nu, \pi\rangle$ *is defined and* $\mathcal{A}^*\langle \nu, \pi\rangle = \nu$ *then* $\pi = \varepsilon$, *and*

$\mathcal{R} = \emptyset$.

If Sign is a PS87 signature then I write $\mathrm{CFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$ for the set of PS87 concrete feature structures under Sign, and $\mathfrak{C}_1 \preceq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{C}_2$ for PS87 concrete feature structure $\mathfrak{C}_2$ subsumes PS87 concrete feature structure $\mathfrak{C}_1$ under Sign.

82 Examples. *The concrete feature structure of example 53 is a PS87 concrete feature structure, but the concrete feature structures of examples 54 and 55 are not.*

83 Definition. *For each PS87 signature* Sign,

$\mathfrak{A}$ *is a* ***PS87 abstract feature structure*** *under* Sign *iff*

$\mathfrak{A} = \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle \in \mathrm{AFS}_{\mathrm{Sign}}$,

$\mathcal{D}$ *is finite, and*

$\mathcal{I} = \emptyset$.

If Sign is a PS87 signature then I write $\mathrm{AFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$ for the set of PS87 abstract feature structures under Sign, and $\mathfrak{A}_1 \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{A}_2$ for PS87 abstract feature structure $\mathfrak{A}_2$ subsumes PS87 abstract feature structure $\mathfrak{A}_1$ under Sign.

84 Proposition. *For each PS87 signature* Sign, $\langle \mathrm{AFS}^{\mathrm{PS87}}_{\mathrm{Sign}}, \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}}\rangle$ *is a partial order.*

85 Proposition. *For each PS87 signature* Sign,

for each $\mathfrak{C} \in \mathrm{CFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$, *for some* $\mathfrak{A} \in \mathrm{AFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$, $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{A} \in \mathrm{AFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$, *for some* $\mathfrak{C} \in \mathrm{CFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$, $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{C}_1 \in \mathrm{CFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$, *for each* $\mathfrak{C}_2 \in \mathrm{CFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$,

$\mathfrak{C}_1 \preceq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{C}_2$ *iff* $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_1) \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_2)$, *and*

for each $\mathfrak{C}_1 \in \mathrm{CFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$, *for each* $\mathfrak{C}_2 \in \mathrm{CFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$,

$\mathfrak{C}_1 \preceq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{C}_2$ *and* $\mathfrak{C}_2 \preceq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{C}_1$ *iff* $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_1) = \mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_2)$.

86 Definition. *For each PS87 signature* Sign,

$\mathfrak{D}$ *is a* ***PS87 disjunctive feature structure*** *under* Sign *iff*

$\mathfrak{D} \in \mathrm{DFS}_{\mathrm{Sign}}$, *and*

$\mathfrak{D} \subseteq \mathrm{AFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$.

If Sign is a PS87 signature then I write $\mathrm{DFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$ for the set of PS87 disjunctive feature structures under Sign, and $\mathfrak{D}_1 \ll^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{D}_2$ for PS87 disjunctive feature structure $\mathfrak{D}_2$ subsumes PS87 disjunctive feature structure $\mathfrak{D}_1$ under Sign.

87 Example. *The disjunctive feature structure of example 75 is a PS87 disjunctive feature structure.*

88 Proposition. *For each PS87 signature* Sign, $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$ *is a partial order.*

89 Lemma. *For each PS87 signature* Sign, $\langle \mathrm{AFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \sqsubseteq^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$ *is upward Noetherian.*

90 Definition. *For each PS87 signature* Sign,

$\mathrm{Close}^{\mathrm{PS87}}_{\mathsf{Sign}}$ *is the total function from* $\mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$ *to* $\mathrm{Pow}(\mathrm{AFS}^{\mathrm{PS87}}_{\mathsf{Sign}})$ *such that*

for each $\mathfrak{D} \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$, $\mathrm{Close}^{\mathrm{PS87}}_{\mathsf{Sign}}(\mathfrak{D}) = \left\{ \mathfrak{A}' \in \mathrm{AFS}^{\mathrm{PS87}}_{\mathsf{Sign}} \,\middle|\, \textit{for some } \mathfrak{A} \in \mathfrak{D},\ \mathfrak{A}' \sqsubseteq^{\mathrm{PS87}}_{\mathsf{Sign}} \mathfrak{A} \right\}$.

91 Lemma. *For each PS87 signature* Sign,

$\mathrm{Close}^{\mathrm{PS87}}_{\mathsf{Sign}}$ *is a bijection from* $\mathfrak{D}^{\mathrm{PS87}}_{\mathsf{Sign}}$ *to* $\mathrm{Down}(\langle \mathrm{AFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \sqsubseteq^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle)$, *and*

for each $\mathfrak{D}_1 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$, *for each* $\mathfrak{D}_2 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

$\mathfrak{D}_1 \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \mathfrak{D}_2$ *iff* $\mathrm{Close}^{\mathrm{PS87}}_{\mathsf{Sign}}(\mathfrak{D}_1) \subseteq \mathrm{Close}^{\mathrm{PS87}}_{\mathsf{Sign}}(\mathfrak{D}_2)$.

92 Proposition. *For each PS87 signature* Sign, $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$ *is a Heyting algebra.*

If Sign is a PS87 signature then I write

$\top^{\mathrm{PS87}}_{\mathsf{Sign}}$ for the top in $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$,

$\bot^{\mathrm{PS87}}_{\mathsf{Sign}}$ for the bottom in $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$,

$\vee^{\mathrm{PS87}}_{\mathsf{Sign}}$ for the join in $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$,

$\wedge^{\mathrm{PS87}}_{\mathsf{Sign}}$ for the meet in $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$,

$\rightarrow^{\mathrm{PS87}}_{\mathsf{Sign}}$ for the relative pseudocomplement in $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$,

$\neg^{\mathrm{PS87}}_{\mathsf{Sign}}$ for the pseudocomplement in $\langle \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}, \ll^{\mathrm{PS87}}_{\mathsf{Sign}} \rangle$.

93 Thesis. *For each natural language* L, *for some PS87 signature* Sign,

L *can be construed as partial information, and*

for each $\mathfrak{D} \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

if $\mathfrak{D}$ *is true of* L *then* $\mathfrak{D}$ *models* L.

3.2 A Shortcoming

[Pollard and Sag 1987] puts feature structures to two distinct purposes. [Pollard and Sag 1987] uses feature structures sometimes to model partial information:

> The fundamental nature of linguistic information remains a mystery. We cannot look at a piece of linguistic information under a microscope; we aren't even sure whether to look inside human brains or somewhere else. Still, although we aren't sure what linguistic information is, we can still theorize about it by trying to say what it is like. To be more precise, we can try to construct formal *models* that reflect certain interesting aspects of the things we are studying. . . .
>
> In the past several years, theoretical and computational linguists working within a number of distinct research traditions have found it useful to construct formal models of linguistic information. . . . the formal objects of choice in information-based linguistics are things known as *feature structures*. [Pollard and Sag 1987, page 27]

sometimes to describe linguistic entities:

> In our applications, we will think of feature structures as partial descriptions of signs (or sign tokens) and other linguistic objects which occur as parts of signs.
>
> [Pollard and Sag 1987, page 28]

and sometimes to do both:

> Intuitively, a feature structure is just an information-bearing object that describes or represents another thing by specifying *values* for various *attributes* of the described thing; we think of the feature structure as providing partial information about the thing described.
>
> [Pollard and Sag 1987, page 28]

This last passage makes clear the nature of this dual purpose. Briefly stated, [Pollard and Sag 1987] has in mind a triangular relationship between feature structures, partial information and linguistic objects. Partial information is *about* linguistic objects, feature structures *model* partial information, and feature structures *describe* linguistic objects, such that

a feature structure describes a linguistic object

iff the feature structure models partial information about the linguistic object.

Indeed, most of [Pollard and Sag 1987]'s intuitions about the modelling of partial information are expressed in terms of the description of linguistic objects. For example, concerning abstract feature structure subsumption and unification,[3] [Pollard and Sag 1987] says that

[3] [Pollard and Sag 1987] calls two abstract feature structures *unifiable* iff they have a greatest lower bound, and calls the

> In general, if feature structure A is at least as informative as (i.e. equally informative as or more informative than) feature structure B, we often say A *extends* (or is an *extension* of) B, and write A $\preceq$ B. Alternatively we say that B *subsumes* A. The relation dual to $\preceq$ is usually called *subsumption*. This latter locution is connected with the obvious fact that any object that could be appropriately described by A could also be appropriately described by B: the less information you have about a thing, the wider the range of possibilities for what that thing might be. [Pollard and Sag 1987, page 30]

> Suppose that A and B are features structures that appropriately describe a certain object X. Then we can think of A and B as expressing *true propositions* about X. Now as we saw above, since A and B describe the same thing they can be unified, and A $\wedge$ B [the unification of A and B] also describes X. Conversely, if we know that C describes X and that C is A $\wedge$ B, then we also know that A and B describe X. So A $\wedge$ B expresses a true proposition about X if and only if both A and B do. Thus the unification of feature structures corresponds to the *logical conjunction* of the propositions that they express. Using the notation of propositional logic: if P is the proposition about X expressed by A and Q is the proposition about X expressed by B, then A $\wedge$ B expresses the proposition P AND Q. [Pollard and Sag 1987, page 40]

Though both passages concern abstract feature structures, every indication is that [Pollard and Sag 1987] intends the subsumption and unification of disjunctive feature structures to behave similarly, where the unification of disjunctive feature structures A and B is the greatest lower bound of A and B in the Heyting algebra of disjunctive feature structures. For example, while discussing the lattice of disjunctive feature structures, [Pollard and Sag 1987] offers the following caution.

> this lattice is "upside down" from the point of view of many computer scientists who are accustomed to identifying "bigger" with "more information". But it is "rightside up" from the point of view of standard logic, in the sense that $\wedge$ [meet] and $\vee$ [join] correspond to conjunction and disjunction respectively. The best mnemonic here is that one feature structure is bigger [higher in the lattice] than another if it appropriately describes a bigger set of objects. [Pollard and Sag 1987, page 42, footnote 10]

Moreover, [Pollard and Sag 1987] motivates the construction of the Heyting algebra of disjunctive feature structures with the observation that

> there are other operations on logical propositions besides conjunction, e.g. disjunction (P OR Q), implication (P IMPLIES Q), and negation (NOT P). Correspondingly, linguistic information can be of a disjunctive, implicative (i.e. conditional), or negative nature. Thus we need to augment our system of [abstract] feature structures to include these notions. [Pollard and Sag 1987, page 40]

Concerning disjunction, [Pollard and Sag 1987] says that

> It often comes about that we know of some linguistic object X that it is appropriately described by *either* A *or* B, but we don't know which. This information is called the *disjunction* of A and B, written A $\vee$ B. [Pollard and Sag 1987, page 41]

[Pollard and Sag 1987, footnote 9, pages 41 and 42] then goes on to define the disjunction of A and B to be the join of A and B in the Heyting algebra of feature structures. Concerning implication and negation, [Pollard and Sag 1987] says that

> We introduced feature structures (in the generalized sense) in order to model disjunctive information. As it turns out, they are sufficient for modelling conditional (implicative) and negative information as well. We begin with implicative information; negative information...is a special case of conditional information. It often comes about that we know of a linguistic object X that *if* it satisfies description A, then it must also satisfy description B. [Pollard and Sag 1987, page 43]

[Pollard and Sag 1987, footnote 12, pages 44] then goes on to define A implies B to be the relative pseudocomplement of A and B in the Heyting algebra of feature structures.

If by the denotation of a feature structure I mean the set of linguistic objects that the feature structure describes then the foregoing quoted passages and others constitute a conjecture concerning the soundness with respect to denotation of disjunctive feature structure join, meet, relative pseudocomplement and pseudocomplement that can be summarised as follows.

94 Conjecture. *For each PS87 signature* Sign,

1: *for each* $\mathfrak{D}_1 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$, *for each* $\mathfrak{D}_2 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$,

the denotation of $\mathfrak{D}_1 \vee^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{D}_2 =$ *the denotation of* $\mathfrak{D}_1 \cup$ *the denotation of* $\mathfrak{D}_2$,

greatest lower bound the *unification* of the feature structures.

2: *for each* $\mathfrak{D}_1 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$, *for each* $\mathfrak{D}_2 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

the denotation of $\mathfrak{D}_1 \wedge^{\mathrm{PS87}}_{\mathsf{Sign}} \mathfrak{D}_2 =$ *the denotation of* $\mathfrak{D}_1 \cap$ *the denotation of* $\mathfrak{D}_2$,

3: *for each* $\mathfrak{D}_1 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$, *for each* $\mathfrak{D}_2 \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

the denotation of $\mathfrak{D}_1 \rightarrow^{\mathrm{PS87}}_{\mathsf{Sign}} \mathfrak{D}_2$

$=$ (*the set complement of the denotation of* $\mathfrak{D}_1$) $\cup$ *the denotation of* $\mathfrak{D}_2$, *and*

4: *for each* $\mathfrak{D} \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

the denotation of $\neg^{\mathrm{PS87}}_{\mathsf{Sign}} \mathfrak{D} =$ *the set complement of the denotation of* $\mathfrak{D}$.

A formal study of this conjecture requires a formal definition of denotation, but any such formalisation risks misrepresenting the intentions of [Pollard and Sag 1987], since [Pollard and Sag 1987] says very little about denotation. Nonetheless, several points can still be made using only an informal notion of description and the observation that a disjunctive feature structure describes an object iff some abstract feature structure in the disjunctive feature structure describes the object.

Conjecture 1 is true. This is unsurprising, since [Pollard and Sag 1987] built the disjunctive feature structures specifically to enable the disjunction of partial information. More surprisingly, conjecture 2 is false. However, the intuitions of [Pollard and Sag 1987] permitting, there are a number of ways to modify the [Pollard and Sag 1987] formalism to make conjecture 2 true. More seriously, conjectures 3 and 4 are false, and without known remedy. Since pseudocomplement is a special case of relative pseudocomplement, it suffices to show that conjecture 4 is false.

95 Proposition. *For some PS87 signature* Sign, *for some* $\mathfrak{D} \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

the denotation of $\neg^{\mathrm{PS87}}_{\mathsf{Sign}} \mathfrak{D} \neq$ *the set complement of the denotation of* $\mathfrak{D}$.

Proof. Suppose that

$\mathrm{Sort} = \{\mathsf{person}, \mathsf{town}\}$,

$\trianglelefteq = \{\langle \mathsf{person}, \mathsf{person}\rangle, \langle \mathsf{town}, \mathsf{town}\rangle\}$,

$\mathrm{Feat} = \{\textsc{home}, \textsc{work}\}$, and

$\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}\rangle$.

Then

Sign is a PS87 signature.

Suppose that

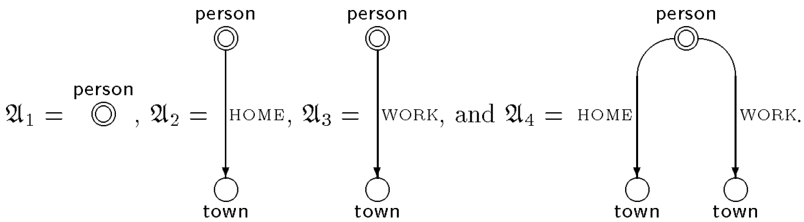
Adam lives in Tübingen but works in Stuttgart.

Then

for each $\mathfrak{A} \in \mathrm{AFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

$\mathfrak{A}$ describes Adam iff ($\mathfrak{A}_1 = \mathfrak{A}$ or $\mathfrak{A}_2 = \mathfrak{A}$ or $\mathfrak{A}_3 = \mathfrak{A}$ or $\mathfrak{A}_4 = \mathfrak{A}$),

where

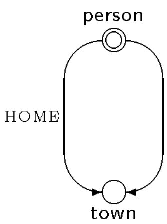


Thus,

for each $\mathfrak{D} \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$,

$\mathfrak{D}$ describes Adam iff ($\mathfrak{A}_1 \in \mathfrak{D}$ or $\mathfrak{A}_2 \in \mathfrak{D}$ or $\mathfrak{A}_3 \in \mathfrak{D}$ or $\mathfrak{A}_4 \in \mathfrak{D}$). ...(*)

Suppose that

$\mathfrak{A}_0 =$ 

, and

$\mathfrak{D} = \{\mathfrak{A}_0\}$

Then

$\mathfrak{A}_0 \in \mathrm{AFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$, and

$\mathfrak{D} \in \mathrm{DFS}^{\mathrm{PS87}}_{\mathsf{Sign}}$.

On one hand,

$\mathfrak{A}_0$ does not describe Adam.

Therefore,

$\mathfrak{D}$ does not describe Adam. ...(1)

On the other,

$\neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ describes Adam

$\Longrightarrow \mathfrak{A}_1 \in \neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ or $\mathfrak{A}_2 \in \neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ or $\mathfrak{A}_3 \in \neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ or $\mathfrak{A}_4 \in \neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ by (*)

$\Longrightarrow \mathfrak{D} \ll^{\mathrm{PS87}}_{\mathrm{Sign}} \neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ since $\mathfrak{A}_0 \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{A}_1$, $\mathfrak{A}_0 \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{A}_2$, $\mathfrak{A}_0 \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{A}_3$ and $\mathfrak{A}_0 \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{A}_4$

$\Longrightarrow \mathfrak{D} = \mathfrak{D} \wedge^{\mathrm{PS87}}_{\mathrm{Sign}} \neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$

$\Longrightarrow \mathfrak{D} = \bot^{\mathrm{PS87}}_{\mathrm{Sign}}$

$\Longrightarrow$ false. since $\bot = \emptyset$

Therefore,

$\neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ does not describe Adam. ...(2)

Therefore,

neither $\mathfrak{D}$ nor $\neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$ describes Adam. by (1) and (2)

Therefore

the denotation of $\neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D} \neq$ the set complement of the denotation of $\mathfrak{D}$.

■

3.3 Conversion Table

Lecture notes	[Pollard and Sag 1987]
Signatures	
PS87 signature; Sign	*no counterpart*; *no counterpart*
sort; Sort	type; *no counterpart*
σ_1 subsumes σ_2; $\sigma_2 \trianglelefteq \sigma_1$	σ_1 subsumes σ_2; $\sigma_2 \preceq \sigma_1$
feature; Feat	attribute; *no counterpart*
Concrete/Abstract feature structures	
[Pollard and Sag 1987] *does not distinguish between concrete and abstact feature structures*	
PS87 abstract feature structure; $\mathrm{AFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$	feature structure, basic feature structure; *no counterpart*
$\mathfrak{A}_1$ subsumes $\mathfrak{A}_2$; $\mathfrak{A}_2 \sqsubseteq^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{A}_1$	$\mathfrak{A}_1$ subsumes $\mathfrak{A}_2$, $\mathfrak{A}_2$ extends $\mathfrak{A}_1$; $\mathfrak{A}_2 \preceq \mathfrak{A}_1$
the greatest lower bound of $\mathfrak{A}_1$ and $\mathfrak{A}_2$	the unification of $\mathfrak{A}_1$ and $\mathfrak{A}_2$
abstraction function; $\mathrm{Abst}_{\mathrm{Sign}}$	*no counterpart*; *no counterpart*
Disjunctive feature structures	
PS87 disjunctive feature structure; $\mathrm{DFS}^{\mathrm{PS87}}_{\mathrm{Sign}}$	feature structure, generalized feature structure; *no counterpart*
$\mathfrak{D}_1$ subsumes $\mathfrak{D}_2$; $\mathfrak{D}_2 \ll^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{D}_1$	$\mathfrak{D}_1$ subsumes $\mathfrak{D}_2$; $\mathfrak{D}_2 \preceq \mathfrak{D}_1$
the top; $\top^{\mathrm{PS87}}_{\mathrm{Sign}}$	the top; $\top$
the bottom; $\bot^{\mathrm{PS87}}_{\mathrm{Sign}}$	the bottom; $\bot$
the least upper bound of $\mathfrak{D}_1$ and $\mathfrak{D}_2$; $\mathfrak{D}_1 \vee^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{D}_2$	the disjunction of $\mathfrak{D}_1$ and $\mathfrak{D}_2$; $\mathfrak{D}_1 \vee \mathfrak{D}_2$
the greatest lower bound of $\mathfrak{D}_1$ and $\mathfrak{D}_2$; $\mathfrak{D}_1 \wedge^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{D}_2$	the unification of $\mathfrak{D}_1$ and $\mathfrak{D}_2$, the conjunction of $\mathfrak{D}_1$ and $\mathfrak{D}_2$; $\mathfrak{D}_1 \wedge \mathfrak{D}_2$
the relative pseudocomplement of $\mathfrak{D}_1$ and $\mathfrak{D}_2$; $\mathfrak{D}_1 \rightarrow^{\mathrm{PS87}}_{\mathrm{Sign}} \mathfrak{D}_2$	the relative pseudocomplement of $\mathfrak{D}_1$ and $\mathfrak{D}_2$; $\mathfrak{D}_1 \Rightarrow \mathfrak{D}_2$
the pseudocomplement of $\mathfrak{D}$; $\neg^{\mathrm{PS87}}_{\mathrm{Sign}}\mathfrak{D}$	the pseudocomplement of $\mathfrak{D}$; $\neg\mathfrak{D}$

4 The [Pollard 1989] Formalism

4.1 The Formalism

The [Pollard 1989] formalism – also found in [Pollard and Moshier 1990] and [Moshier and Pollard 1993] – can be summarised by the following slogans:

A natural language can be identified with partial information, for each sort, about the objects of that sort in the natural language.

Partial information can be modelled by a certain kind of disjunctive feature structure.

The disjunctive feature structures constitute a lattice.

A disjunctive feature structure can be constructed from given abstract feature structures using the operations of the lattice.

Thus, for the [Pollard 1989] formalism, a grammar of a natural language is essentially an assignment to each sort of a recipe for using the operations of a lattice to construct, from given abstract feature structures, a disjunctive feature structure that models partial information about the objects of that sort in the natural language.

96 Definition. Sign *is a* ***Pollard signature*** *iff*

Sign *is a signature* $\langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}\rangle$,

$\langle \mathrm{Sort}, \trianglelefteq\rangle$ *is an upper semilattice with a top,*

Sort *is finite, and*

Feat *is finite.*

97 Example. *The signature of example 51 is a Pollard signature.*

98 Definition. *For each Pollard signature* Sign*,*

$\mathfrak{C}$ *is a* ***Pollard concrete feature structure*** *under* Sign *iff*

$\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R}\rangle \in \mathrm{CFS}_{\mathrm{Sign}}$,

$\mathcal{A}$ *is finite, and*

$\mathcal{R} = \emptyset$.

If Sign is a Pollard signature then I write $\mathrm{CFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$ for the set of Pollard concrete feature structures under Sign, and $\mathfrak{C}_1 \preceq^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{C}_2$ for Pollard concrete feature structure $\mathfrak{C}_2$ subsumes Pollard concrete feature structure $\mathfrak{C}_1$ under Sign.

99 Examples. *The concrete feature structures of examples 53 and 54 are Pollard concrete feature structures, but the concrete feature structure of example 55 is not.*

100 Definition. *For each Pollard signature* Sign*,*

$\mathfrak{A}$ *is a* ***Pollard abstract feature structure*** *under* Sign *iff*

$\mathfrak{A} = \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle \in \mathrm{AFS}_{\mathrm{Sign}}$,

$\mathrm{Quo}(\langle \mathcal{D}, \mathcal{E}\rangle)$ *is finite, and*

$\mathcal{I} = \emptyset$.

If Sign is a Pollard signature then I write $\mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$ for the set of Pollard abstract feature structures under Sign, and $\mathfrak{A}_1 \sqsubseteq^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{A}_2$ for Pollard abstract feature structure $\mathfrak{A}_2$ subsumes Pollard abstract feature structure $\mathfrak{A}_1$ under Sign.

101 Proposition. *For each Pollard signature* Sign, $\langle \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}, \sqsubseteq^{\mathrm{Poll}}_{\mathrm{Sign}}\rangle$ *is a partial order.*

102 Proposition. *For each Pollard signature* Sign*,*

for each $\mathfrak{C} \in \mathrm{CFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$, *for some* $\mathfrak{A} \in \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$, $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{A} \in \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$, *for some* $\mathfrak{C} \in \mathrm{CFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$, $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{C}_1 \in \mathrm{CFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$, *for each* $\mathfrak{C}_2 \in \mathrm{CFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$,

$\mathfrak{C}_1 \preceq^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{C}_2$ *iff* $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_1) \sqsubseteq^{\mathrm{Poll}}_{\mathrm{Sign}} \mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_2)$, *and*

for each $\mathfrak{C}_1 \in \mathrm{CFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$, *for each* $\mathfrak{C}_2 \in \mathrm{CFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$,

$\mathfrak{C}_1 \preceq^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{C}_2$ *and* $\mathfrak{C}_2 \preceq^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{C}_1$ *iff* $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_1) = \mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_2)$.

103 Definition. *For each Pollard signature* Sign*,*

$\mathfrak{D}$ *is a* ***Pollard disjunctive feature structure*** *under* Sign *iff*

$\mathfrak{D} \in \mathrm{DFS}_{\mathrm{Sign}}$, *and*

$\mathfrak{D} \subseteq \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$.

If Sign is a Pollard signature then I write $\mathrm{DFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$ for the set of Pollard disjunctive feature structures under Sign, and $\mathfrak{D}_1 \ll^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{D}_2$ for Pollard disjunctive feature structure $\mathfrak{D}_2$ subsumes Pollard disjunctive feature structure $\mathfrak{D}_1$ under Sign.

104 Example. *The disjunctive feature structure of example 75 is a Pollard disjunctive feature structure.*

105 Proposition. *For each Pollard signature* Sign*,* $\langle \mathrm{DFS}^{\mathrm{Poll}}_{\mathrm{Sign}}, \ll^{\mathrm{Poll}}_{\mathrm{Sign}} \rangle$ *is a partial order.*

106 Lemma. *For each Pollard signature* Sign $= \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$*, for each* $\sigma_1 \in \mathrm{Sort}$*, for each* $\sigma_2 \in \mathrm{Sort}$*,*
if $\{\sigma_1, \sigma_2\}$ *has a lower bound in* $\langle \mathrm{Sort}, \trianglelefteq \rangle$
then $\{\sigma_1, \sigma_2\}$ *has a greatest lower bound in* $\langle \mathrm{Sort}, \trianglelefteq \rangle$*.*

107 Lemma. *For each Pollard signature* Sign*, for each* $\mathfrak{A}_1 \in \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$*, for each* $\mathfrak{A}_2 \in \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$*,*
if $\{\mathfrak{A}_1, \mathfrak{A}_2\}$ *has a lower bound in* $\langle \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}, \sqsubseteq^{\mathrm{Poll}}_{\mathrm{Sign}} \rangle$
then $\{\mathfrak{A}_1, \mathfrak{A}_2\}$ *has a greatest lower bound in* $\langle \mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}, \sqsubseteq^{\mathrm{Poll}}_{\mathrm{Sign}} \rangle$*.*

108 Proposition. *For each Pollard signature* Sign*,* $\langle \mathrm{DFS}^{\mathrm{Poll}}_{\mathrm{Sign}}, \ll^{\mathrm{Poll}}_{\mathrm{Sign}} \rangle$ *is a lattice.*

109 Definition. *For each Pollard signature* Sign $= \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$*,*
G *is a* ***Pollard grammar*** *under* Sign *iff*
G *is a total function from* Sort *to* $\mathrm{DFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$*.*

110 Thesis. *For each natural language* L*, for some Pollard signature* Sign $= \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$*,*
L *can be construed as a set* $\{L_\sigma \mid \sigma \in \mathrm{Sort}\}$*,*
for each $\sigma \in \mathrm{Sort}$*,* L_σ *can be construed as partial information about objects of sort* σ*, and*
for each Pollard grammar G *under* Sign*,*
if G *is true of* L *then for each* $\sigma \in \mathrm{Sort}$*,* $G(\sigma)$ *models* L_σ*.*

4.2 Conversion Table

Lecture notes	[Pollard 1989]
Signatures	
Pollard signature; Sign	*no counterpart*; *no counterpart*
sort; Sort	sort, sort symbol; *Sorts*
σ_1 subsumes σ_2; $\sigma_2 \trianglelefteq \sigma_1$	σ_1 subsumes σ_2; $\sigma_1 \sqsubseteq \sigma_2$
feature; Feat	attribute, attribute label; *Labels*
Concrete feature structures	
Pollard concrete feature structure; $\mathrm{CFS}_{\mathrm{Sign}}$	ordinary feature structure; *OF*
$\mathfrak{C}_1$ subsumes $\mathfrak{C}_2$; $\mathfrak{C}_2 \preceq_{\mathrm{Sign}} \mathfrak{C}_1$	$\mathfrak{C}_1$ subsumes $\mathfrak{C}_2$; $\mathfrak{C}_1 \sqsubseteq_{OF} \mathfrak{C}_2$
Abstract feature structures	
Pollard abstract feature structure; $\mathrm{AFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$	abstract ordinary feature structure; *AOF*
$\mathfrak{A}_1$ subsumes $\mathfrak{A}_2$; $\mathfrak{A}_2 \sqsubseteq^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{A}_1$	$\mathfrak{A}_1$ subsumes $\mathfrak{A}_2$; $\mathfrak{A}_1 \sqsubseteq_{AOF} \mathfrak{A}_2$
abstraction function; $\mathrm{Abst}_{\mathrm{Sign}}$	*no counterpart*; *no counterpart*
Disjunctive feature structures	
Pollard disjunctive feature structure; $\mathrm{DFS}^{\mathrm{Poll}}_{\mathrm{Sign}}$	disjunctive abstract ordinary feature structure; *DAOF*
$\mathfrak{D}_1$ subsumes $\mathfrak{D}_2$; $\mathfrak{D}_2 \ll^{\mathrm{Poll}}_{\mathrm{Sign}} \mathfrak{D}_1$	$\mathfrak{D}_1$ subsumes $\mathfrak{D}_2$; $\mathfrak{D}_1 \sqsubseteq_{DAOF} \mathfrak{D}_2$
Grammars	
Pollard grammar	grammar

5 The [Carpenter 1992] Formalism

5.1 The Formalism

The [Carpenter 1992] formalism can be summarised by the following slogans:

A natural language can be identified with partial information, for each sort, about the objects of that sort in the natural language.

Partial information can be modelled by a certain kind of disjunctive feature structure.

Each disjunctive feature structure constitutes an interpretation of a formal language of descriptions (the Kasper and Rounds logic of [Rounds and Kasper 1986]).

A disjunctive feature structure can be indicated by interpreting a description.

Thus, for the [Carpenter 1992] formalism, a grammar of a natural language is essentially an assignment to each sort of a description that indicates a disjunctive feature structure that models partial information about the objects of that sort in the natural language.

111 Definition. *Sign is a **Carpenter signature** iff Sign is a Pollard signature.*

I assume that none of the symbols False, $\approx$, $\not\approx$, :, $\wedge$, $\vee$, [and] is a Carpenter sort or feature.

112 Definition. *For each Carpenter signature Sign,*

*$\mathfrak{C}$ is a **Carpenter concrete feature structure** under Sign iff*

$\mathfrak{C} = \langle \mathcal{N}, \hat{\nu}, \mathcal{A}, \mathcal{L}, \mathcal{R}\rangle \in \mathrm{CFS}_{\mathrm{Sign}}$, and

$\mathcal{A}$ is finite.

If Sign is a Carpenter signature then I write $\mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$ for the set of Carpenter concrete feature structures under Sign, and $\mathfrak{C}_1 \preceq^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{C}_2$ for Carpenter concrete feature structure $\mathfrak{C}_2$ subsumes Carpenter concrete feature structure $\mathfrak{C}_1$ under Sign.

113 Examples. *The concrete feature structures of examples 53, 54 and 55 are all Carpenter concrete feature structures.*

114 Definition. *For each Carpenter signature Sign,*

*$\mathfrak{A}$ is a **Carpenter abstract feature structure** under Sign iff*

$\mathfrak{A} = \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle \in \mathfrak{A}_{\mathrm{Sign}}$, and

$\mathrm{Quo}\langle \mathcal{D}, \mathcal{E}\rangle$ is finite.

If Sign is a Carpenter signature then I write $\mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$ for the set of Carpenter abstract feature structures under Sign, and $\mathfrak{A}_1 \sqsubseteq^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{A}_2$ for Carpenter abstract feature structure $\mathfrak{A}_2$ subsumes Carpenter abstract feature structure $\mathfrak{A}_1$ under Sign.

115 Proposition. *For each Carpenter signature Sign, $\langle \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}, \sqsubseteq^{\mathrm{Carp}}_{\mathrm{Sign}}\rangle$ is a partial order.*

116 Proposition. *For each Carpenter signature Sign,*

for each $\mathfrak{C} \in \mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$, for some $\mathfrak{A} \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$, $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{A} \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$, for some $\mathfrak{C} \in \mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$, $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}) = \mathfrak{A}$,

for each $\mathfrak{C}_1 \in \mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$, for each $\mathfrak{C}_2 \in \mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\mathfrak{C}_1 \preceq^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{C}_2$ iff $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_1) \sqsubseteq^{\mathrm{Carp}}_{\mathrm{Sign}} \mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_2)$, and

for each $\mathfrak{C}_1 \in \mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$, for each $\mathfrak{C}_2 \in \mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\mathfrak{C}_1 \preceq^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{C}_2$ and $\mathfrak{C}_2 \preceq^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{C}_1$ iff $\mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_1) = \mathrm{Abst}_{\mathrm{Sign}}(\mathfrak{C}_2)$.

117 Proposition. *For each Carpenter signature $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}\rangle$, for each $\pi \in \mathrm{Feat}^*$, for each $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$,*

if $\pi \in \mathcal{D}$ then $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle @\pi \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$.

118 Definition. *For each Carpenter signature Sign,*

*$\mathfrak{D}$ is a **Carpenter disjunctive feature structure** under Sign iff*

$\mathfrak{D} \in \mathrm{DFS}_{\mathrm{Sign}}$, and

$\mathfrak{D} \subseteq \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$.

If Sign is a Carpenter signature then I write $\mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$ for the set of Carpenter disjunctive feature structures under Sign, and $\mathfrak{D}_1 \ll^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{D}_2$ for Carpenter disjunctive feature structure $\mathfrak{D}_2$ subsumes Carpenter disjunctive feature structure $\mathfrak{D}_1$ under Sign.

119 Example. *The disjunctive feature structures of examples 75 and 77 are Carpenter disjunctive feature structures.*

120 Proposition. *For each Carpenter signature Sign, $\langle \mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}}, \ll^{\mathrm{Carp}}_{\mathrm{Sign}}\rangle$ is a partial order.*

121 Definition. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$,

$\mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$ *is the smallest set such that*

$\mathrm{False} \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

for each $\sigma \in \mathrm{Sort}$, $\sigma \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$, $\pi_1 \approx \pi_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$, $\pi_1 \not\approx \pi_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

for each $\pi \in \mathrm{Feat}^*$, *for each* $\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, $\pi{:}\,\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, $[\delta_1 \vee \delta_2] \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *and*

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, $[\delta_1 \wedge \delta_2] \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$.

If Sign is a Carpenter signature then I call each member of $\mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$ a **Carpenter description** under Sign.

122 Example. *If* Sign *is the signature of example 51 then*

$\mathrm{False} \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\mathsf{wife} \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\approx\, \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\text{SPOUSE} \not\approx \text{FRIEND} \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$[\mathsf{wife} \;\wedge\; \text{SPOUSE} \not\approx \text{FRIEND}] \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\text{FRIEND}{:}\,[\mathsf{wife} \;\wedge\; \text{SPOUSE} \not\approx \text{FRIEND}] \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

etc.

123 Definition. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$,

$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}$ *is the total function from* $\mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$ *to* $\mathrm{Pow}(\mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}})$ *such that*

$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\mathrm{False}) = \emptyset$,

for each $\sigma \in \mathrm{Sort}$,

$$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\sigma) = \left\{ \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}} \,\middle|\, \mathcal{F}(\varepsilon) \trianglelefteq \sigma \right\},$$

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

$$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\pi_1 \approx \pi_2) = \left\{ \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}} \,\middle|\, \langle \pi_1, \pi_2 \rangle \in \mathcal{E} \right\},$$

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

$$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\pi_1 \not\approx \pi_2) = \left\{ \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}} \,\middle|\, \langle \pi_1, \pi_2 \rangle \in \mathcal{I} \right\},$$

for each $\pi \in \mathrm{Feat}^*$, *for each* $\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\pi{:}\,\delta) = \left\{ \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}} \,\middle|\, \pi \in \mathcal{D} \text{ and } \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle @\pi \in \mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta) \right\},$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}([\delta_1 \vee \delta_2]) = \mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta_1) \cup \mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta_2), \text{ and}$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

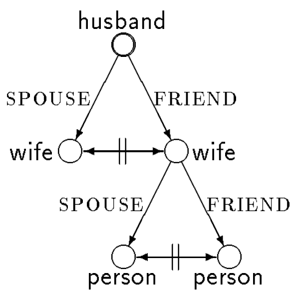
$$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}([\delta_1 \wedge \delta_2]) = \mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta_1) \cap \mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta_2).$$

If Sign is a Carpenter signature then I call $\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta)$ the **Carpenter abstract satisfaction function** under Sign.

124 Example. *If* Sign *is the signature of example 51 then*

$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\mathrm{False}) = \emptyset$,

$\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\approx) = \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$,



$\in \mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\text{FRIEND}{:}\,[\mathsf{wife} \;\wedge\; \text{SPOUSE} \not\approx \text{FRIEND}])$,

etc.

125 Definition. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$,

$\mathrm{DSats}^{\mathrm{Carp}}_{\mathrm{Sign}}$ *is the total function from* $\mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$ *to* $\mathrm{Pow}(\mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}})$ *such that*

for each $\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$$\mathrm{DSats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta) = \left\{ \mathfrak{D} \in \mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}} \,\middle|\, \mathfrak{D} \subseteq \mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta) \right\}.$$

If Sign is a Carpenter signature then I call $\mathrm{DSats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta)$ the **Carpenter disjunctive satisfaction function** under Sign.

126 Definition. *For each Carpenter signature* Sign, *for each* $\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\mathfrak{D} \in \mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\mathfrak{D}$ *is a* ***Carpenter most general satisfier*** *of* δ *under* Sign *iff*

for each $\mathfrak{D}' \in \mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

$\mathfrak{D}' \in \mathrm{DSats}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta)$ *iff* $\mathfrak{D}' \ll^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{D}$.

127 Lemma. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$, *for each* $\sigma_1 \in \mathrm{Sort}$, *for each* $\sigma_2 \in \mathrm{Sort}$,

if $\{\sigma_1, \sigma_2\}$ *has a lower bound in* $\langle \mathrm{Sort}, \trianglelefteq \rangle$

then $\{\sigma_1, \sigma_2\}$ *has a greatest lower bound in* $\langle \mathrm{Sort}, \trianglelefteq \rangle$.

128 Lemma. *For each Carpenter signature* Sign, *for each* $\mathfrak{A}_1 \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\mathfrak{A}_2 \in \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

if $\{\mathfrak{A}_1, \mathfrak{A}_2\}$ *has a lower bound in* $\langle \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}, \sqsubseteq^{\mathrm{Carp}}_{\mathrm{Sign}} \rangle$

then $\{\mathfrak{A}_1, \mathfrak{A}_2\}$ *has a greatest lower bound in* $\langle \mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}, \sqsubseteq^{\mathrm{Carp}}_{\mathrm{Sign}} \rangle$.

129 Proposition. *For each Carpenter signature* Sign, $\langle \mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}}, \ll^{\mathrm{Carp}}_{\mathrm{Sign}} \rangle$ *is a lattice.*

130 Lemma. *For each Carpenter signature* Sign,

False *has a unique most general satisfier under* Sign.

131 Lemma. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$, *for each* $\sigma \in \mathrm{Sort}$

σ *has a unique most general satisfier under* Sign.

132 Lemma. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$, *for each* $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

$\pi_1 \approx \pi_2$ *has a unique most general satisfier under* Sign.

133 Lemma. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$, *for each* $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

$\pi_1 \not\approx \pi_2$ *has a unique most general satisfier under* Sign.

134 Lemma. *For each Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$, *for each* $\pi \in \mathrm{Feat}^*$, *for each* $\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

if δ *has a unique most general satisfier under* Sign

then $\pi{:}\,\delta$ *has a unique most general satisfier under* Sign.

135 Lemma. *For each Carpenter signature* Sign, *for each* $\delta_1 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

if δ_1 *has a unique most general satisfier under* Sign, *and*

δ_2 *has a unique most general satisfier under* Sign

then $[\delta_1 \vee \delta_2]$ *has a unique most general satisfier under* Sign.

136 Lemma. *For each Carpenter signature* Sign, *for each* $\delta_1 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

if δ_1 *has a unique most general satisfier under* Sign, *and*

δ_2 *has a unique most general satisfier under* Sign

then $[\delta_1 \wedge \delta_2]$ *has a unique most general satisfier under* Sign.

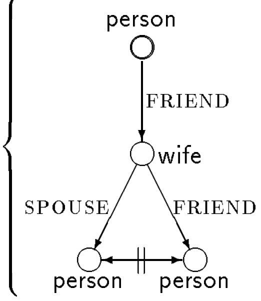
137 Proposition. *For each Carpenter signature* Sign, *for each* $\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$,

there is a unique most general satisfier of δ *under* Sign.

If Sign is a Carpenter signature and $\delta \in \mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$ then I write $\mathrm{MGSat}^{\mathrm{Carp}}_{\mathrm{Sign}}(\delta)$ for the unique most general satisfier of δ under Sign.

138 Example. *If* Sign *is the signature of example 51 then*

$$\mathrm{MGSat}^{\mathrm{Carp}}_{\mathrm{Sign}}(\textsc{friend}\colon[\mathsf{wife}\ \wedge\ \textsc{spouse} \not\approx \textsc{friend}]) = \left\{ \quad \right\}.$$



139 Definition. *For each Carpenter signature* Sign,
G is a **Carpenter grammar** *under* Sign *iff*
G is a total function from Sort *to* $\mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$.

140 Thesis. *For each natural language L, for some Carpenter signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}\rangle$,
L can be construed as a set $\{L_\sigma \mid \sigma \in \mathrm{Sort}\}$,
for each $\sigma \in \mathrm{Sort}$, L_σ *can be construed as partial information about objects of sort* σ, *and*
for each Carpenter grammar G under Sign,
if G is true of L then for each $\sigma \in \mathrm{Sort}$, $\mathrm{MGSat}^{\mathrm{Carp}}_{\mathrm{Sign}}(G(\sigma))$ *models* L_σ.

5.2 Conversion Table

Lecture notes	[Carpenter 1992]
Signatures	
Carpenter signature; Sign	inheritance hierarchy (*partial counterpart*); *no counterpart*
sort; Sort	type; Type
σ_1 subsumes σ_2; $\sigma_2 \trianglelefteq \sigma_1$	σ_1 subsumes σ_2; $\sigma_1 \sqsubseteq \sigma_2$
feature; Feat	feature; Feat
Concrete feature structures	
Carpenter concrete feature structure; $\mathrm{CFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$	feature structure; $\mathcal{F}$
$\mathfrak{C}_1$ subsumes $\mathfrak{C}_2$; $\mathfrak{C}_2 \preceq^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{C}_1$	$\mathfrak{C}_1$ subsumes $\mathfrak{C}_2$; $\mathfrak{C}_1 \sqsubseteq \mathfrak{C}_2$
Abstract feature structures	
Carpenter abstract feature structure; $\mathrm{AFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$	abstract feature structure; *no counterpart*
$\mathfrak{A}_1$ subsumes $\mathfrak{A}_2$; $\mathfrak{A}_2 \sqsubseteq \mathfrak{A}_1$	$\mathfrak{A}_1$ subsumes $\mathfrak{A}_2$; *no full counterpart*
abstraction function; Abst	*no counterpart*; *Abs*
Disjunctive feature structures	
Carpenter disjunctive feature structure; $\mathrm{DFS}^{\mathrm{Carp}}_{\mathrm{Sign}}$	*no counterpart*; *no counterpart*
$\mathfrak{D}_1$ subsumes $\mathfrak{D}_2$; $\mathfrak{D}_2 \ll^{\mathrm{Carp}}_{\mathrm{Sign}} \mathfrak{D}_1$	*no counterpart*; *no counterpart*
Descriptions and Satisfiers	
Carpenter description; $\mathrm{Desc}^{\mathrm{Carp}}_{\mathrm{Sign}}$	description; Desc
Carpenter abstract satisfaction function; $\mathrm{ASats}^{\mathrm{Carp}}_{\mathrm{Sign}}$	satisfaction relation (*partial counterpart*); $\models$ (*partial counterpart*)
Carpenter disjunctive satisfaction function; $\mathrm{DSats}^{\mathrm{Carp}}_{\mathrm{Sign}}$	*no counterpart*; *no counterpart*
Carpenter most general satisfier; $\mathrm{MGSat}^{\mathrm{Carp}}_{\mathrm{Sign}}$	disjunctive most general satisfiers; *MGSats*
Grammars	
Carpenter grammar	constraint system

6 The [King 1989] Formalism

6.1 The Formalism

The [King 1989] formalism – developed in [King 1994] and [King 1997] – can be summarised by the following slogans:

A natural language can be identified with the set of objects in the natural language.

Each set of objects constitutes an interpretation of a formal language of descriptions (the Speciate Re-entrant Logic of [King 1989]).

A set of objects can be indicated by interpreting a set of descriptions.

Thus, for the [King 1989] formalism, a grammar of a natural language is essentially a set of descriptions that indicates the set of objects in the natural language.

141 Definition. Sign *is a* ***King signature*** *iff*

Sign *is a triple* $\langle$Spec, Feat, Appr$\rangle$,

Spec *is a set,*

Feat *is a set, and*

Appr *is a total function from* Spec $\times$ Feat *to* Pow(Spec).

If Sign $=$ $\langle$Spec, Feat, Appr$\rangle$ is a King signature then I call each member of Spec a **species** under Sign, each member of Feat a **feature** under Sign, and Appr the **appropriateness function** under Sign. I assume that none of the symbols :, $\sim$, $\approx$, $\neg$, $\wedge$, $\vee$, $\rightarrow$, [and] is a King species or feature.

142 Example. *If*

Spec $= \{\mathsf{single}, \mathsf{wife}, \mathsf{husband}\}$,

Feat $= \{\textsc{spouse}, \textsc{friend}\}$,

Appr *is the total function from* Spec $\times$ Feat *to* Pow(Spec) *such that*

Appr$\langle\mathsf{single}, \textsc{spouse}\rangle = \emptyset$,

Appr$\langle\mathsf{single}, \textsc{friend}\rangle = \{\mathsf{single}, \mathsf{wife}, \mathsf{husband}\}$,

Appr$\langle\mathsf{wife}, \textsc{spouse}\rangle = \{\mathsf{husband}\}$,

Appr$\langle\mathsf{wife}, \textsc{friend}\rangle = \{\mathsf{single}, \mathsf{wife}, \mathsf{husband}\}$,

Appr$\langle\mathsf{husband}, \textsc{spouse}\rangle = \{\mathsf{wife}\}$, *and*

Appr$\langle\mathsf{husband}, \textsc{friend}\rangle = \{\mathsf{single}, \mathsf{wife}, \mathsf{husband}\}$, *and*

Sign $=$ $\langle$Spec, Feat, Appr$\rangle$

then Sign *is a King signature.*

143 Definition. *For each King signature* Sign $=$ $\langle$Spec, Feat, Appr$\rangle$,

I *is a* ***King interpretation*** *under* Sign *iff*

I *is a triple* $\langle U, S, F\rangle$,

U *is a set,*

S *is a total function from* U *to* Spec,

F *is a total function from* Feat *to the set of partial functions from* U *to* U, *and*

for each $\varphi \in$ Feat, *for each* $\upsilon \in U$,

$F(\varphi)(\upsilon)$ *is defined iff* Appr$\langle S(\upsilon), \varphi\rangle \neq \emptyset$, *and*

if $F(\varphi)(\upsilon)$ *is defined then* $S(F(\varphi)(\upsilon)) \in$ Appr$\langle S(\upsilon), \varphi\rangle$.

If Sign is a King signature and $I = \langle U, S, F\rangle$ is a King interpretation under Sign then I call U the **universe** under I, each member of U an **object** under I, S the **species interpretation function** under I, and F the **feature interpretation function** under I, and I write Uni(I) for U.

144 Example. *Suppose that*

Anne and Adam are a married couple, as are Beth and Bill, but Cath and Carl are unmarried.

Suppose further that

Anne and Adam are best friends, as are Cath and Carl, but Beth's best friend is Cath and Bill's best friend is Adam.

From this scenario, we can construct a King interpretation $I = \langle U, S, F\rangle$ *of the King signature* Sign $=$ $\langle$Spec, Feat, Appr$\rangle$ *of example 142 as follows. Let the objects be the people Anne, Adam, Beth, Bill, Cath and Carl. Thus,*

$U = \{\text{Anne}, \text{Adam}, \text{Beth}, \text{Bill}, \text{Cath}, \text{Carl}\}$.

The objects can be of many different species, including married person, woman, or person whose name begins with 'C'. However, the natural interpretations of the species in Spec *are captured by the total function* S *from* U *to* Spec *such that*

$S(\mathrm{Anne}) = \mathsf{wife}$,

$S(\mathrm{Adam}) = \mathsf{husband}$,

$S(\mathrm{Beth}) = \mathsf{wife}$,

$S(\mathrm{Bill}) = \mathsf{husband}$,

$S(\mathrm{Cath}) = \mathsf{single}$, *and*

$S(\mathrm{Carl}) = \mathsf{single}$.

Similarly, the objects can have many different attributes, including their spouse, their name, or their age. However, the natural interpretations of the features in Feat *are captured by the total function* F *from* Feat *to the set of partial functions from* U *to* U *such that*

$F(\textsc{spouse})$ *is the partial function from* U *to* U *such that*

$F(\textsc{spouse})(\mathrm{Anne}) = \mathrm{Adam}$,

$F(\textsc{spouse})(\mathrm{Adam}) = \mathrm{Anne}$,

$F(\textsc{spouse})(\mathrm{Beth}) = \mathrm{Bill}$,

$F(\textsc{spouse})(\mathrm{Bill}) = \mathrm{Beth}$,

$F(\textsc{spouse})(\mathrm{Cath})$ *is undefined, and*

$F(\textsc{spouse})(\mathrm{Carl})$ *is undefined, and*

$F(\textsc{friend})$ *is the partial function from* U *to* U *such that*

$F(\textsc{friend})(\mathrm{Anne}) = \mathrm{Adam}$,

$F(\textsc{friend})(\mathrm{Adam}) = \mathrm{Anne}$,

$F(\textsc{friend})(\mathrm{Beth}) = \mathrm{Cath}$,

$F(\textsc{friend})(\mathrm{Bill}) = \mathrm{Adam}$,

$F(\textsc{friend})(\mathrm{Cath}) = \mathrm{Carl}$, *and*

$F(\textsc{friend})(\mathrm{Carl}) = \mathrm{Cath}$.

Certain features are inappropriate for certain species, in that no object of that species should have that feature. For example, no object of species single *should have the feature* $\textsc{spouse}$. Appr *records this condition by setting* $\mathrm{Appr}\langle\mathsf{single}, \textsc{spouse}\rangle = \emptyset$, *and* S *and* F *meet this condition, since no object (Cath and Carl) of species* single *has the feature* $\textsc{spouse}$.

Certain other features are appropriate for certain species, in that each object of that species should have that feature. For example, each object of species wife *should have the feature* $\textsc{spouse}$. Appr *records this condition by setting* $\mathrm{Appr}\langle\mathsf{wife}, \textsc{spouse}\rangle \neq \emptyset$, *and* S *and* F *meet this condition, since each object (Anne and Beth) of species* wife *has the feature* $\textsc{spouse}$. *Moreover, if a feature* φ *is appropriate for a species* σ *then there are appropriate species for the value of* φ *on* σ, *in that the value of feature* φ *on an object of species* σ *must be an object of some or other prescribed species. For example, the value of feature* $\textsc{spouse}$ *on an object of species* wife *must be an object of species* $\mathsf{husband}$. *Notice that there may be more than one prescribed species: since the best friend of a wife may be married or single, the value of feature* $\textsc{friend}$ *on an object of species* wife *must be an object of species* single, wife *or* $\mathsf{husband}$. Appr *records these conditions by setting* $\mathrm{Appr}\langle\mathsf{wife}, \textsc{spouse}\rangle = \{\mathsf{husband}\}$ *and* $\mathrm{Appr}\langle\mathsf{wife}, \textsc{friend}\rangle = \{\mathsf{single}, \mathsf{wife}, \mathsf{husband}\}$, *and* S *and* F *meet these conditions, since Anne's spouse is an object (Adam) of species* $\mathsf{husband}$, *Beth's spouse is an object (Bill) of species* $\mathsf{husband}$, *Anne's best friend is an object (Adam) of species* $\mathsf{husband}$, *and Beth's best friend is an object (Cath) of species* single.

In light of the foregoing, $I = \langle U, S, F\rangle$ *is a King interpretation under* Sign.

Notice two points. Firstly, each object is of a unique species. No object is of no species, and no object is of more than one species. For example, I cannot construct a King signature for our scenario in which

$\mathrm{Spec} = \{\mathsf{male}, \mathsf{single}\}$

and interpret these symbols in the natural way, since some objects (Carl) are both male and single, while other objects (Anne and Beth) are neither male nor single. Secondly, the species and features can only be interpreted in such a way that a given feature is appropriate for all or no objects of a given species. It cannot be that some objects of a given species have a given feature, while others do not. For example, I cannot construct a King signature for our scenario in which

$\mathrm{Spec} = \{\mathsf{male}, \mathsf{female}\}$ *and* $\mathrm{Feat} = \{\textsc{spouse}\}$

and interpret these symbols in the natural way, since some objects (Anne and Beth) are female and have a spouse, while other objects (Cath) are also female but have no spouse.

145 Definition. *For each King signature* $\mathrm{Sign} = \langle\mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr}\rangle$,

$\mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$ *is the smallest set such that*

$: \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, *and*

for each $\tau \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\varphi \in \mathrm{Feat}$, $\tau\varphi \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$.

If Sign is a King signature then I call each member of $\mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$ a **King term** under Sign.

146 Example. *If* Sign *is the King signature of example 142 then*

$: \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{spouse} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{friend} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{spouse spouse} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{spouse friend} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{friend spouse} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{friend friend} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{spouse spouse spouse} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{spouse spouse friend} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{spouse friend spouse} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$:\textsc{spouse friend friend} \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

etc.

147 Definition. *For each King signature* $\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr} \rangle$, *for each King interpretation* $I = \langle U, S, F \rangle$ *under* Sign,

T^{King}_{I} *is the total function from* $\mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$ *to the set of partial functions from* U *to* U *such that*

$T^{\mathrm{King}}_{I}(:)$ *is the identity function from* U *to* U, *and*

for each $\tau \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\varphi \in \mathrm{Feat}$, *for each* $\upsilon \in U$,

$T^{\mathrm{King}}_{I}(\tau\varphi)(\upsilon)$ *is defined iff* $T^{\mathrm{King}}_{I}(\tau)(\upsilon)$ *and* $F(\varphi)(T^{\mathrm{King}}_{I}(\tau)(\upsilon))$ *are defined, and*

if $T^{\mathrm{King}}_{I}(\tau\varphi)$ *is defined then* $T^{\mathrm{King}}_{I}(\tau\varphi)(\upsilon) = F(\varphi)(T^{\mathrm{King}}_{I}(\tau)(\upsilon))$.

If Sign is a King signature and I is a King interpretation of Sign then I call T^{King}_{I} the **King term interpretation function** under I.

148 Example. *If* Sign *is the King signature of example 142 and* I *is the King interpretation under* Sign *of example 144 then*

$T^{\mathrm{King}}_{I}(:)(\mathrm{Beth}) = \mathrm{Beth}$,

$T^{\mathrm{King}}_{I}(:\textsc{spouse})(\mathrm{Beth}) = \mathrm{Bill}$,

$T^{\mathrm{King}}_{I}(:\textsc{friend})(\mathrm{Beth}) = \mathrm{Cath}$,

$T^{\mathrm{King}}_{I}(:\textsc{spouse spouse})(\mathrm{Beth}) = \mathrm{Beth}$,

$T^{\mathrm{King}}_{I}(:\textsc{spouse friend})(\mathrm{Beth}) = \mathrm{Adam}$,

$T^{\mathrm{King}}_{I}(:\textsc{friend spouse})(\mathrm{Beth})$ *is undefined*,

$T^{\mathrm{King}}_{I}(:\textsc{friend friend})(\mathrm{Beth}) = \mathrm{Carl}$,

$T^{\mathrm{King}}_{I}(:\textsc{spouse spouse spouse})(\mathrm{Beth}) = \mathrm{Bill}$,

$T^{\mathrm{King}}_{I}(:\textsc{spouse spouse friend})(\mathrm{Beth}) = \mathrm{Cath}$,

$T^{\mathrm{King}}_{I}(:\textsc{spouse friend spouse})(\mathrm{Beth}) = \mathrm{Anne}$,

$T^{\mathrm{King}}_{I}(:\textsc{spouse friend friend})(\mathrm{Beth}) = \mathrm{Anne}$,

etc.

149 Definition. *For each King signature* $\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr} \rangle$,

$\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$ *is the smallest set such that*

for each $\tau \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\sigma \in \mathrm{Spec}$, $\tau \sim \sigma \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

for each $\tau_1 \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\tau_2 \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, $\tau_1 \approx \tau_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

for each $\delta \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, $\neg\delta \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, $[\delta_1 \vee \delta_2] \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, $[\delta_1 \wedge \delta_2] \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *and*

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, $[\delta_1 \rightarrow \delta_2] \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$.

If Sign is a King signature then I call each member of $\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$ a **King description** under Sign, and each subset of $\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$ a **King theory** under Sign.

150 Example. *If* Sign *is the King signature of example 142 and* I *is the King interpretation under* Sign *of example 144 then*

$:\mathrm{FRIEND}\ \mathrm{SPOUSE} \sim \mathsf{wife} \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
$:\mathrm{SPOUSE} \approx :\mathrm{FRIEND} \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
$[:\mathrm{FRIEND}\ \mathrm{SPOUSE} \sim \mathsf{wife}\ \wedge\ \neg :\mathrm{SPOUSE} \approx :\mathrm{FRIEND}] \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
$:\approx: \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
$\neg :\approx: \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
$\{:\mathrm{FRIEND}\ \mathrm{SPOUSE} \sim \mathsf{wife}, \neg :\mathrm{SPOUSE} \approx :\mathrm{FRIEND}\} \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
$\{:\mathrm{FRIEND}^i \sim \mathsf{single} \mid i \text{ is a natural number}\} \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
$\emptyset \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
etc.

151 Definition. *For each King signature* $\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr} \rangle$, *for each King interpretation* $I = \langle U, S, F \rangle$ *under* Sign,

D^{King}_I *is the total function from* $\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$ *to* $\mathit{Pow}(U)$ *such that*

for each $\tau \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\sigma \in \mathrm{Spec}$,

$$D^{\mathrm{King}}_I(\tau \sim \sigma) = \left\{ \upsilon \in U \,\middle|\, \begin{array}{l} T^{\mathrm{King}}_I(\tau)(\upsilon) \text{ is defined, and} \\ S(T^{\mathrm{King}}_I(\tau)(\upsilon)) = \sigma \end{array} \right\},$$

for each $\tau_1 \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\tau_2 \in \mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$D^{\mathrm{King}}_I(\tau_1 \approx \tau_2) = \left\{ \upsilon \in U \,\middle|\, \begin{array}{l} T^{\mathrm{King}}_I(\tau_1)(\upsilon) \text{ is defined,} \\ T^{\mathrm{King}}_I(\tau_2)(\upsilon) \text{ is defined, and} \\ T^{\mathrm{King}}_I(\tau_1)(\upsilon) = T^{\mathrm{King}}_I(\tau_2)(\upsilon) \end{array} \right\},$$

for each $\delta \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$D^{\mathrm{King}}_I(\neg\delta) = U \setminus D^{\mathrm{King}}_I(\delta),$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$D^{\mathrm{King}}_I([\delta_1 \vee \delta_2]) = D^{\mathrm{King}}_I(\delta_1) \cup D^{\mathrm{King}}_I(\delta_2),$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$D^{\mathrm{King}}_I([\delta_1 \wedge \delta_2]) = D^{\mathrm{King}}_I(\delta_1) \cap D^{\mathrm{King}}_I(\delta_2), \text{ and}$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$D^{\mathrm{King}}_I([\delta_1 \rightarrow \delta_2]) = (U \setminus D^{\mathrm{King}}_I(\delta_1)) \cup D^{\mathrm{King}}_I(\delta_2).$$

If Sign is a King signature and I is a King interpretation under Sign then I call D^{King}_I the **King description denotation function** under I.

152 Example. *If* Sign *is the King signature of example 142 and* I *is the King interpretation under* Sign *of example 144 then*

$D^{\mathrm{King}}_I(:\mathrm{FRIEND}\ \mathrm{SPOUSE} \sim \mathsf{wife}) = \{\mathrm{Anne}, \mathrm{Bill}\}$,
$D^{\mathrm{King}}_I(:\mathrm{SPOUSE} \approx :\mathrm{FRIEND}) = \{\mathrm{Anne}, \mathrm{Adam}\}$,
$D^{\mathrm{King}}_I([:\mathrm{FRIEND}\ \mathrm{SPOUSE} \sim \mathsf{wife}\ \wedge\ \neg :\mathrm{SPOUSE} \approx :\mathrm{FRIEND}]) = \{\mathrm{Bill}\}$,
$D^{\mathrm{King}}_I(:\approx:) = U$,
$D^{\mathrm{King}}_I(\neg :\approx:) = \emptyset$,
etc.

153 Definition. *For each King signature* Sign, *for each King interpretation* I *under* Sign,

Θ^{King}_I *is the total function from* $\mathit{Pow}(\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}})$ *to* $\mathit{Pow}(\mathrm{Uni}(I))$ *such that*

for each $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, $\Theta^{\mathrm{King}}_I(\theta) = \{\upsilon \in \mathrm{Uni}(I) \mid \text{for each } \delta \in \theta,\ \upsilon \in D^{\mathrm{King}}_I(\delta)\}$.

If Sign is a King signature and I is a King interpretation of Sign then I call Θ^{King}_I the **King theory denotation function** under I.

154 Example. *If* Sign *is the King signature of example 142 and* I *is the King interpretation under* Sign *of example 144 then*

$\Theta^{\mathrm{King}}_I(\{:\mathrm{FRIEND}\ \mathrm{SPOUSE} \sim \mathsf{wife}, \neg :\mathrm{SPOUSE} \approx :\mathrm{FRIEND}\}) = \{\mathrm{Bill}\}$,
$\Theta^{\mathrm{King}}_I(\{:\mathrm{FRIEND}^i \sim \mathsf{single} \mid i \text{ is a natural number}\}) = \{\mathrm{Cath}, \mathrm{Carl}\}$,
$\Theta^{\mathrm{King}}_I(\emptyset) = U$,
etc.

155 Definition. *For each King signature* Sign, *for each King interpretation* I *under* Sign, *for each* $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

I ***satisfies*** θ under Sign *iff* $\Theta^{\mathrm{King}}_I(\theta) \neq \emptyset$, *and*

I ***models*** θ under Sign *iff* $\Theta^{\mathrm{King}}_I(\theta) = \mathrm{Uni}(I)$.

156 Example. *If* Sign *is the King signature of example 142 and* I *is the King interpretation under* Sign *of example 144 then*

I *satisfies* $\{:\textsc{friend spouse} \sim \mathsf{wife}, \neg:\textsc{spouse} \approx :\textsc{friend}\}$, *and*

I *models* $\emptyset$.

157 Definition. *For each King signature* Sign, *for each King interpretation* I *under* Sign, *for each* $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

I ***exhaustively models*** θ under Sign *iff*

I models θ under Sign, *and*

for each King interpretation I' under Sign, *for each* $\theta' \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

if I' models θ under Sign *and I' satisfies θ' under* Sign *then I satisfies θ under* Sign.

158 Example. *Suppose that*

$\mathrm{Spec} = \{\mathsf{person}, \mathsf{town}\}$,

$\mathrm{Feat} = \{\textsc{home}, \textsc{work}\}$,

Appr *is the total function from* Spec $\times$ Feat *to* Pow(Spec) *such that*

$\mathrm{Appr}\langle \mathsf{person}, \textsc{home}\rangle = \{\mathsf{town}\}$,

$\mathrm{Appr}\langle \mathsf{person}, \textsc{work}\rangle = \{\mathsf{town}\}$,

$\mathrm{Appr}\langle \mathsf{town}, \textsc{home}\rangle = \emptyset$, *and*

$\mathrm{Appr}\langle \mathsf{town}, \textsc{work}\rangle = \emptyset$, *and*

$\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr}\rangle$.

Then Sign *is a King signature. Suppose that*

$\theta = \{[:\sim \mathsf{person} \ \rightarrow \ \neg:\textsc{home} \approx :\textsc{work}]\}$.

Then $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$. *Suppose that*

Adam lives in Tübingen but works in Stuttgart.

More formally, suppose that

$U = \{\mathrm{Adam}, \mathrm{Tuebingen}, \mathrm{Stuttgart}\}$,

S is a total function from U to Spec *such that*

$S(\mathrm{Adam}) = \mathsf{person}$,

$S(\mathrm{Tuebingen}) = \mathsf{town}$, *and*

$S(\mathrm{Stuttgart}) = \mathsf{town}$,

F is the total function from Feat *to the set of partial functions from U to U such that*

$F(\textsc{home})$ is the partial function from U to U such that

$F(\textsc{home})(\mathrm{Adam}) = \mathrm{Tuebingen}$, *and*

$F(\textsc{work})$ is the partial function from U to U such that

$F(\textsc{work})(\mathrm{Adam}) = \mathrm{Stuttgart}$, *and*

$I = \langle U, S, F\rangle$.

Then

I is a King interpretation under Sign, *and*

I exhaustively models θ under S.

159 Definition. *For each King signature* Sign,

$\langle v_1, I_1\rangle$ and $\langle v_2, I_2\rangle$ are ***indiscernible*** under Sign *iff*

I_1 is a King interpretation under Sign,

I_2 is a King interpretation under Sign,

$v_1 \in \mathrm{Uni}(I_1)$,

$v_2 \in \mathrm{Uni}(I_2)$, *and*

for each $\delta \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$v_1 \in D^{\mathrm{King}}_{I_1}(\delta)$ *iff* $v_2 \in D^{\mathrm{King}}_{I_2}(\delta)$.

160 Definition. *For each King signature* Sign,
I_1 ***simulates*** I_2 *under* Sign *iff*
I_1 *is a King interpretation under* Sign,
I_2 *is a King interpretation under* Sign, *and*
for each $\upsilon_2 \in \mathrm{Uni}(I_2)$, *for some* $\upsilon_1 \in \mathrm{Uni}(I_1)$,
$\langle \upsilon_1, I_1 \rangle$ *and* $\langle \upsilon_2, I_2 \rangle$ *are indiscernible under* Sign.

161 Proposition. *For each King signature* Sign, *for each King interpretation* I *under* Sign, *for each* $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
I *exhaustively models* θ *under* Sign *iff*
I *models* θ *under* Sign, *and*
for each King interpretation I' *under* Sign,
if I' *models* θ *under* Sign *then* I *simulates* I' *under* Sign.

162 Thesis. *For each natural language* L, *for some King signature* Sign,
L *can be construed as a King interpretation under* Sign, *and*
for each $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
if θ *is true of* L *then* L *exhaustively models* θ *under* Sign.

6.2 Conversion Table

Lecture notes	[King 1989]
Signatures	
King signature; Sign	signature; SIGN
species; Spec	species; SPEC
feature; Feat	attribute; ATTR
appropriateness function; Appr	descendence function; DESC
Interpretations	
King interpretation; I	interpretation; I
universe; object; U, $\mathrm{Uni}(I)$	domain; object; D
species interpretation function; S	species assignment function; S
feature interpretation function; F	attribute interpretation function; A
Terms, Descriptions and Theories	
King term; $\mathrm{Term}^{\mathrm{King}}_{\mathrm{Sign}}$	path; PATH
King term interpretation function; T^{King}_{I}	path interpretation function; P_I
King description; King theory; $\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$	formula; *no counterpart*; FORM
King description denotation function; D^{King}_{I}	denotation; $[\,]_I$
King theory denotation function; $\Theta^{\mathrm{King}}_{I}$	denotation; $[\,]_I$
Models	
models	*no counterpart*
satisfies	*no counterpart*
exhaustively models	*no counterpart*

7 The [Pollard and Sag 1994] Formalism

7.1 The Formalism

The [Pollard and Sag 1994] formalism can be summarised by the following slogans:

> A natural language can be identified with the set of object types in the natural language.
>
> An object type can be modelled by a certain kind of abstract feature structure.
>
> Each set of abstract feature structures constitutes an interpretation of a (loosely specified) formal language of descriptions.
>
> A set of abstract feature structures can be indicated by interpreting a set of descriptions.

Thus, for [Pollard and Sag 1994], a grammar of a natural language is essentially a set of descriptions that indicates a set of abstract feature structures that models the set of object types in the natural language.

[Pollard and Sag 1994] specifies a somewhat complicated signature and interpretation.

> The (finite) set of all sort symbols is assumed to be partially ordered, with sort symbols corresponding to more inclusive types lower in the ordering. For example, the sort *sign* is ordered below the sort *phrase* or *word* because signs include both phrases and words; we say, for example, that *word* is a *subsort* of *sign* and *accusative* is a subsort of *case*.
>
> [Pollard and Sag 1994, page 17]

In addition,

> The sort hierarchy is presented as a taxonomic tree, with the root labelled *object* (the sort of all linguistic entities with which the grammar deals). For each local tree in the hierarchy, the sorts $\sigma_1, \ldots, \sigma_n$, which label the daughters, partition the sort σ, which labels the mother; that is, they are necessarily disjoint subsorts of σ that exhaust σ. For two sorts σ and τ, τ is a *subsort* of σ if and only if it is dominated by σ; sorts that label terminal nodes are called *maximal* (in the sense of maximally informative or maximally specific). A *feature declaration* of the form
>
> $$\sigma\colon \begin{bmatrix} \mathrm{F}_1\ \tau_1 \\ \ldots \\ \mathrm{F}_n\ \tau_n \end{bmatrix}$$
>
> where $\sigma, \tau_1, \ldots, \tau_n$ are sorts and $\mathrm{F}_1, \ldots, \mathrm{F}_n$ are feature labels, signifies that for each $i = 1, \ldots, n$, (1) the feature F_i is appropriate for all objects of sort σ, and (2) for any such object, the value of the F_i feature must be an object of sort τ_i.
>
> If sorts σ_1 and σ_2 bear the declarations $[\mathrm{F}\ \tau_1]$ and $[\mathrm{F}\ \tau_2]$ for the same feature F and σ_2 is a subsort of σ_1, then τ_2 must be a subsort of τ_1. A sort inherits the feature declarations of its supersorts; hence any feature that is defined for a given sort is defined for all of that sort's subsorts. By convention, the features that are declared for a maximal sort (including those inherited from its supersorts) are the only features defined for that sort. A special case is that of *atomic* sorts or simply *atoms*, maximal sorts for which no features are defined.
>
> [Pollard and Sag 1994, pages 395 and 396]

I *think* that the following formalises the foregoing.

163 Definition. Sign *is a* ***PS94 signature*** *iff*

Sign *is a quadruple* $\langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}, \mathrm{Decl} \rangle$,

$\langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat} \rangle$ *is a signature,*

Sort *is finite,*

for each $\sigma_1 \in \mathrm{Sort}$, *for each* $\sigma_2 \in \mathrm{Sort}$, *for each* $\sigma_3 \in \mathrm{Sort}$,

if $\sigma_1 \trianglelefteq \sigma_2$ *and* $\sigma_1 \trianglelefteq \sigma_3$

then $\sigma_2 \trianglelefteq \sigma_3$ *or* $\sigma_3 \trianglelefteq \sigma_2$,

$\langle \mathrm{Sort}, \trianglelefteq \rangle$ *has a top,*

Decl *is a partial function from* $\mathrm{Sort} \times \mathrm{Feat}$ *to* Sort, *and*

for each $\sigma_1 \in \mathrm{Sort}$, *for each* $\sigma_2 \in \mathrm{Sort}$, *for each* $\varphi \in \mathrm{Feat}$,

if $\sigma_1 \trianglelefteq \sigma_2$ *and* $\mathrm{Decl}\langle \sigma_2, \varphi \rangle$ *is defined*

then $\mathrm{Decl}\langle \sigma_1, \varphi \rangle$ *is defined and* $\mathrm{Decl}\langle \sigma_1, \varphi \rangle \trianglelefteq \mathrm{Decl}\langle \sigma_2, \varphi \rangle$.

If $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}, \mathrm{Decl} \rangle$ is a PS94 signature then I call Decl the **feature declaration function** under Sign.

164 Definition. *For each PS94 signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}, \mathrm{Decl} \rangle$, *for each* $\sigma \in \mathrm{Sort}$,

σ *is* ***maximal*** *under* Sign *iff for each* $\sigma' \in \mathrm{Sort}$, *if* $\sigma' \trianglelefteq \sigma$ *then* $\sigma' = \sigma$.

If Sign is a PS94 signature then I write $\mathrm{Max}^{\mathrm{PS94}}_{\mathrm{Sign}}$ for the set of maximal sorts in Sign.

165 Definition. *For each PS94 signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}, \mathrm{Decl}\rangle$,
$\mathfrak{A}$ *is a* ***PS94 feature structure*** *under* Sign *iff*
$\mathfrak{A}$ *is an abstract feature structure* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle$ *under* $\langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}\rangle$
$\mathcal{F}$ *is a total function from* $\mathcal{D}$ *to* $\mathrm{Max}^{\mathrm{PS94}}_{\mathrm{Sign}}$,
for each $\pi \in \mathcal{D}$, *for each* $\varphi \in \mathrm{Feat}$,
$\pi\varphi \in \mathcal{D}$ *iff* $\mathrm{Decl}\langle \mathcal{F}(\pi), \varphi\rangle$ *is defined, and*
if $\pi\varphi \in \mathcal{D}$ *then* $\mathcal{F}(\pi\varphi) \trianglelefteq \mathrm{Decl}\langle \mathcal{F}(\pi), \varphi\rangle$, *and*
for each $\pi_1 \in \mathcal{D}$, *for each* $\pi_2 \in \mathcal{D}$,
if $\langle \pi_1, \pi_2\rangle \notin \mathcal{E}$ *then* $\langle \pi_1, \pi_2\rangle \in \mathcal{I}$.

If Sign is a PS94 signature then I write $\mathrm{FS}^{\mathrm{PS94}}_{\mathrm{Sign}}$ for the set of PS94 feature structures under Sign.

166 Thesis. *For each natural language* L, *for some PS94 signature* Sign,
L *can be construed as a set of object types, and*
for each theory θ,
if θ *is true of* L
then θ *admits all and only the PS94 feature structures that model object types in* L.

7.2 A Completion

Here I expand the [King 1989] formalism to include the notion of object type and show that this expansion provides a possible completion of the [Pollard and Sag 1994] formalism.

167 Definition. *For each PS94 signature* $\mathrm{Sign} = \langle \mathrm{Sort}, \trianglelefteq, \mathrm{Feat}, \mathrm{Decl}\rangle$,
$\mathrm{King}(\mathrm{Sign}) = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Decl}\rangle$, *where*
$\mathrm{Spec} = \mathrm{Max}^{\mathrm{PS94}}_{\mathrm{Sign}}$, *and*
Appr *is the total function from* $\mathrm{Spec} \times \mathrm{Feat}$ *to* $\mathrm{Pow}(\mathrm{Spec})$ *such that for each* $\sigma \in \mathrm{Spec}$, *for each* $\varphi \in \mathrm{Feat}$,
if $\mathrm{Decl}\langle\sigma, \varphi\rangle$ *is undefined then* $\mathrm{Appr}\langle\sigma, \varphi\rangle = \emptyset$, *and*
if $\mathrm{Decl}\langle\sigma, \varphi\rangle$ *is defined then* $\mathrm{Appr}\langle\sigma, \varphi\rangle = \left\{\sigma' \in \mathrm{Spec} \,\middle|\, \sigma' \trianglelefteq \mathrm{Decl}\langle\sigma, \varphi\rangle\right\}$.

168 Proposition. *For each PS94 signature* Sign, $\mathrm{King}(\mathrm{Sign})$ *is a King signature.*

169 Proposition. *For each PS94 signature* Sign, *for each* I,
I *is a PS94 interpretation under* Sign *iff* I *is a King interpretation under* $\mathrm{King}(\mathrm{Sign})$.

170 Definition. *For each King signature* Sign, *for each King interpretation under* Sign, *for each* $\upsilon_1 \in \mathrm{Uni}(I_1)$, *for each* $\upsilon_2 \in \mathrm{Uni}(I_2)$,
$\upsilon_1 \equiv_I \upsilon_2$ *iff* $\langle\upsilon_1, I\rangle$ *and* $\langle\upsilon_2, I\rangle$ *are indiscenible under* Sign.

171 Proposition. *For each King signature* Sign, *for each King interpretation* I *under* Sign,
$\langle I, \equiv_I\rangle$ *is an equivalence relation.*

172 Definition. *For each King signature* Sign, *for each King interpretation* I *under* Sign,
$\mathrm{Type}(I) = \mathrm{Quo}(\langle I, \equiv_I\rangle)$.

If Sign is a King signature and I is a King interpretation under Sign then I call each member of $\mathrm{Type}(I)$ a **type** in I under Sign.

173 Definition. *For each King signature* $\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr}\rangle$,
$\mathfrak{A}$ *is a* ***King feature structure*** *under* Sign *iff*
$\mathfrak{A}$ *is an abstract feature structure* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle$ *under* $\langle \mathrm{Spec}, =, \mathrm{Feat}\rangle$
for each $\pi \in \mathcal{D}$, *for each* $\varphi \in \mathrm{Feat}$,
$\pi\varphi \in \mathcal{D}$ *iff* $\mathrm{Appr}\langle \mathcal{F}(\pi), \varphi\rangle \neq \emptyset$, *and*
if $\pi\varphi \in \mathcal{D}$ *then* $\mathcal{F}(\pi\varphi) \in \mathrm{Appr}\langle \mathcal{F}(\pi), \varphi\rangle$, *and*
for each $\pi_1 \in \mathcal{D}$, *for each* $\pi_2 \in \mathcal{D}$,
if $\langle \pi_1, \pi_2\rangle \notin \mathcal{E}$ *then* $\langle \pi_1, \pi_2\rangle \in \mathcal{I}$.

If Sign is a King signature then I write $\mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}}$ for the set of King feature structures.

174 Proposition. *For each PS94 signature* Sign, *for each* $\mathfrak{A}$,
$\mathfrak{A}$ *is a PS94 feature structure under* Sign *iff* $\mathfrak{A}$ *is a King feature structure under* $\mathrm{King}(\mathrm{Sign})$.

175 Definition. *For each King signature* $\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr}\rangle$,
$\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}$ *is the total function from* $\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$ *to* $\mathrm{Pow}(\mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}})$ *such that*
for each $\pi \in \mathrm{Feat}^*$, *for each* $\sigma \in \mathrm{Spec}$,
$\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(:\pi \sim \sigma) = \left\{\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I}\rangle \in \mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}} \,\middle|\, \mathcal{F}(\pi) = \sigma\right\}$,

for each $\pi_1 \in \mathrm{Feat}^*$, *for each* $\pi_2 \in \mathrm{Feat}^*$,

$$\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(:\pi_1 \approx :\pi_2) = \left\{ \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}} \,\middle|\, \langle \pi_1, \pi_2 \rangle \in \mathcal{E} \right\},$$

for each $\delta \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\neg\delta) = (\mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}} \setminus \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta)),$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}([\delta_1 \vee \delta_2]) = \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta_1) \cup \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta_2),$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}([\delta_1 \wedge \delta_2]) = \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta_1) \cap \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta_2), \text{ and}$$

for each $\delta_1 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\delta_2 \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$$\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}([\delta_1 \rightarrow \delta_2]) = (\mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}} \setminus \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta)) \cup \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta_2).$$

If Sign is a King signature then I call $\mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta)$ the **King satisfaction function** under Sign.

176 Definition. *For each King signature* Sign,

$\mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}$ *is the total function from* $\mathrm{Pow}(\mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}})$ *to* $\mathrm{Pow}(\mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}})$ *such that*

for each $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\mathfrak{A} = \langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}}$,

$\mathfrak{A} \in \mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}(\theta)$ *iff for each* $\delta \in \theta$, *for each* $\pi \in \mathcal{D}$, $\mathfrak{A}@\pi \in \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta)$.

If Sign is a King signature then I call $\mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}(\delta)$ the **King admission function** under Sign.

177 Proposition. *For each King signature* $\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr} \rangle$, *for each* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle \in \mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each* $\pi \in \mathrm{Feat}^*$,

if $\pi \in \mathcal{D}$ *then* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle @\pi \in \mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}}$.

178 Definition. *For each King signature* $\mathrm{Sign} = \langle \mathrm{Spec}, \mathrm{Feat}, \mathrm{Appr} \rangle$, *for each King interpretation* I *under* Sign, *for each* $\upsilon \in \mathrm{Uni}(I)$,

$\mathrm{Abst}^{\mathrm{King}}_I(\upsilon)$ *is the quadruple* $\langle \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{I} \rangle$, *where*

$$\mathcal{D} = \left\{ \pi \in \mathrm{Feat}^* \,\middle|\, T^{\mathrm{King}}_I(:\pi)(\upsilon) \text{ is defined} \right\},$$

$$\mathcal{E} = \left\{ \langle \pi_1, \pi_2 \rangle \in \mathrm{Feat}^* \times \mathrm{Feat}^* \,\middle|\, \begin{array}{l} T^{\mathrm{King}}_I(:\pi_1) \text{ is defined,} \\ T^{\mathrm{King}}_I(:\pi_2) \text{ is defined, and} \\ T^{\mathrm{King}}_I(:\pi_1) = T^{\mathrm{King}}_I(:\pi_2) \end{array} \right\},$$

$$\mathcal{F} = \left\{ \langle \pi, \sigma \rangle \in \mathrm{Feat}^* \times \mathrm{Spec} \,\middle|\, \begin{array}{l} T^{\mathrm{King}}_I(:\pi) \text{ is defined, and} \\ S(T^{\mathrm{King}}_I(:\pi)) = \sigma \end{array} \right\}, \text{ and}$$

$$\mathcal{I} = \left\{ \langle \pi_1, \pi_2 \rangle \in \mathrm{Feat}^* \times \mathrm{Feat}^* \,\middle|\, \begin{array}{l} T^{\mathrm{King}}_I(:\pi_1) \text{ is defined,} \\ T^{\mathrm{King}}_I(:\pi_2) \text{ is defined, and} \\ T^{\mathrm{King}}_I(:\pi_1) \neq T^{\mathrm{King}}_I(:\pi_2) \end{array} \right\}.$$

179 Proposition. *For each King signature* Sign, *for each King interpretation* I *under* Sign,

$\mathrm{Abst}^{\mathrm{King}}_I$ *is a total function from* I *to* $\mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}}$, *and*

for each $\upsilon_1 \in \mathrm{Uni}(I)$, *for each* $\upsilon_2 \in \mathrm{Uni}(I)$,

$\upsilon_1 \equiv^{\mathrm{King}}_I \upsilon_2$ *iff* $\mathrm{Abst}^{\mathrm{King}}_I(\upsilon_1) = \mathrm{Abst}^{\mathrm{King}}_I(\upsilon_2)$.

180 Corollary. *For each King signature* Sign, *for each King interpretation* I *under* Sign,

$\mathrm{Abst}^{\mathrm{King}}_I$ *determines an injection from* $\mathrm{Type}(I)$ *to* $\mathrm{FS}^{\mathrm{King}}_{\mathrm{Sign}}$.

181 Lemma. *For each King signature* Sign, *for each King interpretation* I *under* Sign, *for each* $\upsilon \in \mathrm{Uni}(I)$, *for each* $\delta \in \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,

$\upsilon \in D^{\mathrm{King}}_I(\delta)$ *iff* $\mathrm{Abst}^{\mathrm{King}}_I(\upsilon) \in \mathrm{Sats}^{\mathrm{King}}_{\mathrm{Sign}}(\delta)$.

182 Proposition. *For each King signature* Sign, *for each* $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each King interpretation* I *under* Sign,

I *models* θ *under* Sign

iff $\mathrm{Abst}^{\mathrm{King}}_I$ *is a total function from* $\mathrm{Uni}(I)$ *to* $\mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}(\theta)$.

183 Corollary. *For each King signature* Sign, *for each King interpretation* I *under* Sign,

I *models* θ *under* Sign

iff $\mathrm{Abst}^{\mathrm{King}}_I$ *determines an injection from* $\mathrm{Type}(I)$ *to* $\mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}(\theta)$.

184 Lemma. *For each King signature* Sign, *for each* $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$,
for some King interpretation I *under* Sign,
I models θ under Sign, *and*
$\mathrm{Abst}^{\mathrm{King}}_{I}$ *is a surjection from* $\mathrm{Uni}(I)$ *to* $\mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}(\theta)$.

185 Proposition. *For each King signature* Sign, *for each* $\theta \subseteq \mathrm{Desc}^{\mathrm{King}}_{\mathrm{Sign}}$, *for each King interpretation* I *under* Sign,
I exhaustively models θ under Sign
iff $\mathrm{Abst}^{\mathrm{King}}_{I}$ *is a surjection from* $\mathrm{Uni}(I)$ *to* $\mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}(\theta)$.

186 Corollary. *For each King signature* Sign, *for each King interpretation* I *under* Sign,
I exhaustively models θ under Sign
iff $\mathrm{Abst}^{\mathrm{King}}_{I}$ *determines a bijection from* $\mathrm{Type}(I)$ *to* $\mathrm{Admits}^{\mathrm{King}}_{\mathrm{Sign}}(\theta)$.

7.3 Conversion Table

Lecture notes	[Pollard and Sag 1994]
PS94 signature; Sign	inheritance hierarchy (*partial counterpart*); *no counterpart*
sort; Sort	sort; *no counterpart*
σ_1 subsumes σ_2; $\sigma_2 \trianglelefteq \sigma_1$	σ_1 subsumes σ_2; *no counterpart*
feature; Feat	feature; *no counterpart*
feature declaration function; Decl	feature declaration; *no counterpart*
maximal sort; $\mathrm{Max}^{\mathrm{PS94}}_{\mathrm{Sign}}$	maximal sort; *no counterpart*
PS94 feature structure; $\mathrm{FS}^{\mathrm{PS94}}_{\mathrm{Sign}}$	sorted feature structure; *no counterpart*

8 References

[Carpenter 1992] Bob Carpenter. *The Logic of Typed Feature Structures.* Cambridge Tracts in Theoretical Computer Science, number 32. Cambridge University Press, Cambridge, England. 1992.

[King 1989] Paul J. King. *A Logical Formalism for Head-Driven Phrase Structure Grammar.* Doctoral thesis, Manchester University, Manchester, England. 1989.

[King 1994] Paul J. King. An expanded logical formalism for head-driven phrase structure grammar. Sonderforschungsbereich 340 technical report 59, IBM Deutschland GmbH, Heidelberg, Germany. 1994.

[King 1997] Paul J. King. An expanded logical formalism for head-driven phrase structure grammar. Tentative Title. Book. In preparation. 1997.

[Moshier 1988] M. Andrew Moshier. *Extensions to Unification Grammars for the Description of Programming Languages.* Doctoral thesis, University of Michigan, Ann Arbor, Michigan, USA. 1988.

[Moshier and Pollard 1993] M. Andrew Moshier and Carl J. Pollard. A domain of set-valued feature structures. CLAUS technical report 35, Computerlinguistik, Universität des Saarlandes, Saarbrücken, Germany. 1993.

[Pollard 1989] Carl J. Pollard. Sorts in unification-based grammar and what they mean. Unpublished manuscript. 1989.

[Pollard and Moshier 1990] Carl J. Pollard and M. Andrew Moshier. Unifying partial descriptions of sets. In Philip P. Hanson (editor), *Information, Language and Cognition.* Vancouver Studies in Cognitive Science, number 1. University of British Columbia Press, Vancouver, British Columbia, Canada. 1990.

[Pollard and Sag 1987] Carl J. Pollard and Ivan A. Sag. *Information-Based Syntax and Semantics.* CSLI Lecture Notes, number 13. CSLI, Stanford, California, USA. 1987.

[Pollard and Sag 1994] Carl J. Pollard and Ivan A. Sag. *Head-Driven Phrase Structure Grammar.* University of Chicago Press, Chicago, Illinois, USA. 1994.

[Rounds and Kasper 1986] William C. Rounds and Robert T. Kasper. A complete logical calculus for record structures representing linguistic information. In *Proceedings of the IEEE Symposium on Logic in Computer Science.* 1986.